Carbon tax and sustainable facility location: The role of production technology

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Abstract

Recent studies on facility location under the carbon pricing scheme highlight that increasing the carbon price can ensure meaningful reductions in transport-related greenhouse gas emissions (GHGs). Indeed, when we assume the bill of materials (BOM) as a production technology (i.e. complementary of inputs), a higher carbon price does not change the supply planning because the input proportions are fixed, but it does increase the total transport cost, which pushes the firm to make its location choice more sustainable (lower level of emissions). However, when we account for possible substitution between input quantities, this may cease to hold. In this paper, we propose to revisit the Production-Location Problem (PLP) considering transport-related carbon emission mitigation due to carbon taxation and production technologies that allow complementarity or substitution among input quantities. We first show that cost-minimizing location may differ from carbon emission minimizing location, regardless of the production technology type. We also find that an increased carbon tax may increase carbon emissions when we enable substitution across input quantities. Gradual changes in carbon tax affect the relative delivered prices of inputs (the per-unit procurement and transportation costs of an input compared to the costs of another) such that the firm has an incentive to relocate its facility and substitute among input quantities, leading to new shipping patterns that can generate a higher level of pollution under certain parameters.
Introduction

The different agreements and conferences on climate change, from Kyoto (1997) to Paris (2015), have reflected the growing interest of public authorities to reduce greenhouse gas emissions (UNFCCC (2015), Christoff (2016)). Especially, freight transport accounts for a significant proportion of carbon emissions; it represented approximately 14% of total emissions in 2010, which had increased by 11% over the period of 2000 – 2010 (IPCC (2014)). Thus, it is a major challenge for policy makers to deploy efficient regulations to restrict the carbon emissions from logistic activities of companies (Hoen et al. (2014)). One way to curb the transport-related carbon emissions would be to influence the location choice of companies since the spatial organization of firms affects directly distances traveled by commodities; and transport modes selection and therefore the ecological footprint stemming from commodity shipping (Daniel et al. (1997), Gaigné et al. (2012), Lim et al. (2016)).

Recent studies on facility location under a carbon pricing scheme highlight that increasing the carbon price can lead to firm relocation, which creates better transport environmental performances (see, e.g., Ramudhin et al. (2010), Chaabane et al. (2012), Rezaee et al. (2017)). Such a result is obtained by assuming, either explicitly or implicitly, an assembly supply chain modeled by a bill of materials (BOM). The BOM of a manufactured product is a concept of the late seventies in which all product components are listed in a structured way (Hegge and Wortmann (1991)). It is classically represented by a tree structure and assumes a fixed coefficient for each component in each subset (Pochet and Wolsey (2006)).

1Currently, there are two main types of economic regulatory instruments that could impact transport-related emissions: carbon taxation and carbon emissions trading (Hoen et al. (2013). These regulations are often advocated by international institutions as effective instruments for carbon emissions mitigation (Zhang and Baranzini (2004), Zakeri et al. (2015)).
Consequently, an increase in carbon price will have no effect on the quantities’ supply choices because the demand for each input is fixed \textit{a priori}, but it will increase the total transport costs, thus prompting the firm to relocate to sustainable alternative places (lower level of emissions). In this paper, we argue that the positive environmental result of a higher carbon price may be weakened or may cease to hold when possible substitutions are allowed among the input quantities. The effect of substituting raw materials can be observed in some industrial sectors such as the food industry (e.g. preparation of culinary products), the animal feed industry (e.g. mixture of livestock feed), or the chemical industry (e.g. primary products made at the mine’s exit), where it is often possible for a firm to vary the share of input quantities for regulatory, marketing or even raw material relative prices change reasons (Balakrishnan and Geunes (2000)). In this context, firms may have access to different combinations of input quantities (input mixes) to produce a given level of output. Motivated by the increasing importance to assess the effects of carbon taxes, the objective of this paper is to investigate the facility location problem of a manufacturing firm under a carbon tax policy on transport-related emissions when the input quantities may be substituted. To reach our goal, we reconsider the production-location problem (PLP) (Hale and Moberg (2003)) to allow for the substitution between inputs. The advantage of this type of model is that they enable the simultaneous evaluation of the effects of taxation on both the location decision and the production supply (Hurter and Martinich (1989)). Specifically, the firm incurs sourcing and transportation costs. In addition to these costs, the firm also incurs a carbon tax, based on the distance and the carbon intensity of transportation mode. The relationship between the production level and the quantities of inputs is captured in our framework through two polar cases: (a) the Leontief production function (used as a benchmark case), which is expressly included in the description of the BOM since it implies a fixed (exogenous) proportion to produce the output; and (b) the Cobb Douglass production function under which the two inputs are substitutes. It is worth stressing that the objective of our paper is not to build a comprehensive firm location model under carbon taxation but rather to develop a simple model to clearly identify the mechanisms at work and highlight
the crucial role of production technology and location choice in the evaluation of the carbon taxation policies. These mechanisms are still ambiguous in the literature, as reported by Wu et al. (2017). It is therefore relevant and appropriate to clarify them across a simple structure rather than a general and more complicated structure.

We first show that the cost-minimizing and emissions-minimizing solutions do not always overlap, regardless of the production technology type. Furthermore, in the case of Leontief technology, a higher tax leads to a lower or the same level of emissions, depending on the mode of transportation. When input proportions can be varied under a Cobb-Douglas structure, the tax effects can be ambiguous. To disentangle the various effects at work, we must distinguish between two cases: in the former, the facility location is given (short term), and in the latter, the facility location adjusts to changes in carbon tax (long term). Our analysis relies on the following trade-off. On the one hand, for a given location, an increase in carbon tax reduces transport-related carbon emissions of the supply planning as the relative delivered prices of inputs (the per-unit procurement and transportation costs of an input compared to the costs of another) converge to the relative delivered prices, leading to the lowest level of carbon emissions. In this case, higher carbon tax rates raise total shipping cost and shift the input proportions in the direction that lowers emissions. On the other hand, by affecting the relative delivered prices of inputs, a higher carbon tax can trigger the firm to relocate to re-optimize its cost, inducing a new substitution among input quantities and, thus, a new shipping pattern. Such a relocation can imply a divergence, instead of a convergence, between the relative delivered prices of inputs and the relative delivered prices of inputs minimizing carbon emissions. In other words, when a facility location reacts to changes in carbon tax, a marginal increase in the carbon tax rate can generate a higher level of pollution under certain parameters. This result is sufficient to show that the desirability of carbon taxation is more complex than suggested by its proponents, primarily because this recommendation disregards its impact on the location of production and the substitution among inputs. Accounting for these effects makes the impact of a higher carbon tax more ambiguous because their net effects depend on whether the new spatial pattern is better or
worse from an environmental viewpoint.

The remainder of this paper is organized as follows. In the next section, we present a literature review on sustainable facility location. In section 3 we revisit the PLP formulation. Subsequently, we propose to study the extended PLP model under specific production technology forms. Section 4 focuses on Leontief technology, and section 5 focuses on Cobb-Douglas technology. The last section provides several further conclusions.

2. Literature review

The decision of where to locate a facility is at the intersection of several research disciplines in operations management and economic sciences (Alcacer and Delgado (2016)). It has been extensively studied by making use of, in particular, operations research techniques with a purely economic perspective. The location decision-making was then based on Weberian modeling, whose the main objective is to minimize the logistic costs (Brandeau and Chiu (1989)). The modeling name of "Weberian" honors the work of Weber (1909) on the determination of optimal industrial location (ReVelle and Eiselt (2005)). Since this work, location models have rapidly evolved by incorporating several location types (e.g. production, distribution, suppliers), supply chain levels, mono and multi-periods, single and multiple products, technologies, transport modes, and constraints related to tactical and operational levels (Melo et al. (2009)).

For many years now, researchers have focused increasing attention on improving the sustainability behind facility location decisions by considering not only the economic criteria but also environmental and social criteria. A comprehensive description and classification of the most recent studies and contributions can be found in some reviews (Terouhid et al. (2012), Chen et al. (2014), Eskandarpour et al. (2015)). Environmental aspects are more often considered when determining a sustainable facility location than social aspects. This is usually explained by the toughness of modeling social factors quantitatively in a relevant manner (Jorgensen et al. (2008)). The carbon footprint is generally used as a measurement metric to qualify ecological performance (Hassini et al. (2012)). It corresponds to the amount of carbon dioxide $CO_2$ emitted by a company during all stages of a product’s
life-cycle or in some stages such as production and/or transport. Some of the existing studies that integrated GHG emissions in location modeling generally seek a trade-off between minimizing economic costs (or maximizing profit) and carbon footprint by using a single economic objective model. In this case, the GHG emissions are converted into their monetary equivalent (Elhedhli and Merrick (2012)). In addition to the single objective models, many researchers develop bi-objective models to generate a set of Pareto-optimal solutions (Bouzembrak et al. (2011), Xifeng et al. (2013)). However, few researchers in operations management have incorporated regulations such as carbon tax, carbon emissions trading, and carbon cap in their location models and analyzed the impact of these mechanisms on facility location decisions and the resulting GHG emissions.

Dekker et al. (2012) consider Ramudhin et al. (2010) to be among the first to study the sensitivity of location decisions on carbon market trade. The latter authors consider the problem of sustainable supply chain configuration (e.g. suppliers, subcontracting, distribution, production and transport technology choices) and introduce emission constraints (emissions cap) imposed by law (or by the company itself) with a given carbon price using a Mixed-Integer Linear Programming model (MILP). The analyses of an example from the steel industry reveal that when the emission cap shrinks or the carbon price increases, decision-makers select production schemes and transport modes that are more environmentally friendly. Chaabane et al. (2012) extend the previous model to a Multi-Objective Linear Programming approach (MOLP) and consider the Life-Cycle Assessment (LCA) method to measure carbon emissions. They study the effect of carbon emission allowance and tradable carbon credits and conclude from a case study of the aluminum industry that supply managers have two options. First, when reducing GHG emissions cost through clean technology investments and sustainable supply chain configuration (internal mechanisms) is more expensive than buying carbon credits (external mechanisms), decision-makers buy carbon credits to be compliant with the regulatory limits. Otherwise, they should opt for the greener production and transport strategies. Diabat and Simchi-Levi (2009) consider a MILP model for a supply chain network design with a cap on the amount of CO2 emitted. They use an
experimental example to show that when the cap becomes tighter, the total supply chain cost increases as the firm opens more manufacturing and distribution centers (increasing fixed costs) to reduce transport emissions. Rezaee et al. (2017) propose a discrete location problem with uncertainty on demand and carbon price. They conclude from an office furniture industry case study that a higher carbon price leads to a sustainable supply chain design but not necessarily according to a linear relationship. Cachon (2014) considers the problem of downstream supply chain design (retail store location, density, and size) with the goal of reducing carbon emissions under a carbon price scheme. The findings of this study indicate that the carbon price must reach a very high level for a significant carbon emissions reduction. Wu et al. (2017) investigate the effect of rising carbon prices on off-shoring and near-shoring decisions for a single producer between two distinct regions (north and south) according to different scenarios of production costs, emissions, and the distance that separates the two regions. Turken et al. (2017) address the facility location and capacity acquisition problem under two forms of carbon emissions regulatory mechanisms: a carbon tax and a command-and-control policy that involves taxation after exceeding the emissions limit. They investigate the various effects of the limit on carbon emissions and carbon tax on centralization or decentralization of optimal production network and the resulting emissions. Most of these publications have concluded that a higher carbon prices could lead to reduce carbon emissions resulting from transport, more or less significant depending on the context and nature of the studied problem. However, when assessing the ecological outcomes, the existing literature has neglected one major issue: the role of the production technology type on the relation between carbon tax and location. As noted in the introduction, they did not pay attention to the substitution effect through input quantities. Peeters and Thisse (2000) are among the first to emphasize the importance of a raw material substitution effect on facility location decision. They show that small variations on the elasticity of substitution among input quantities may lead to significant change in the optimal firm location. However, the environmental issues related to the transport of commodities are left aside on their study. Following this literature, we highlight a lack of location models that address the role of
production technology in the relationship between location decision and environmental regulations. Our research contributes to the related literature by investigating the efficiency of carbon tax to reduce transport-related emissions when firms can adjust their locations and have access to different types of production technologies.

3. Model description

In this section, we propose to revisit the PLP model under a carbon tax policy on transport-related carbon emissions. Consider a firm $F$ that must deliver a unique transformed product to one downstream market noted $M$ which has a deterministic demand $q^0$. Downstream demand is assumed to be fixed to focus only on carbon emissions generated by the transport phases. Consider also two different upstream markets, in which each raw material input $i$ provided from a fixed market $S_i$ with an amount noted $s_i$, with $i = 1, 2$. All stakeholders have a fixed and determined location on a one-dimensional linear network space. Without loss of generality, the two input markets are on the left side of the output market $M$ with $S_1$ closer to $M$ than $S_2$. The distances between stakeholders are: $d(S_2, M) = z$, $d(S_2, S_1) = x$ and $d(S_2, F) = y$. The Firm $F$ must define its best location as $y$, which is necessarily on the line segment $[S_2, M]$ (convex hull), as proven by Wendell and Hurter (1973), when the transport costs are non-negative and non-decreasing with distance. Thus, the optimal location can be either between $S_2$ and $S_1$ (case a) or between $S_1$ and $M$ (case b)(see Figure 1). We consider a single facility problem with a mono-product setting to focus on the interrelations between carbon price and firm location choice through the production decisions; besides, these interactions can be clarified and explained more easily with a linear representation and a limited number of suppliers and output markets.

The firm produces the market demand in one shot (static period) without constituting inventories and with an infinite capacity system. The firm has access to different production techniques modeled by a production function $f : R^2 \rightarrow R$ that links the total production level to raw material needs such as:

$$q^0 = f(s_1, s_2).$$
Each unit of raw material $i$ has a mill price $w_i$, a transport cost per unit of distance and per unit of input $t_i$ supported by the firm, and a carbon footprint coefficient per unit of distance and per unit of input $\alpha_i$ from $S_i$ to $F$, with $i = 1, 2$. Each unit of output has a transport cost per unit of distance and per unit of output $t_0$ borne by the firm and a carbon footprint coefficient per unit of distance and per unit of output $\alpha_0$ from $F$ to $M$. Fixed transport costs are not considered since that the firm is at an aggregate strategic level of decision; that is, no economies of scale emerge. In addition, the firm is in a price taker situation and therefore has no influence on output or input prices. Let $d_i$ with $i = 0, 1, 2$ be the distances between the firm location $y$, and the output market and input markets respectively with $d_0 = z - y$, $d_1 = |x - y|$, and $d_2 = y$. Let $\tau$ be the unit carbon price per unit of carbon emissions. The initial rate $\tau$ is often fixed by public authorities. However, its effectiveness depends on its evolution. Thus, it is expected that the carbon tax will increase gradually over time since the damage caused by GHG emissions will have a greater negative impact in the future than currently (Zhang and Baranzini (2004)).

Following Peeters and Thisse (2000), the objective of the firm is to identify the best location $y$ and define its supply strategy, determined by the vector $s = (s_1, s_2)$, to minimize its cost function while respecting its production technology. The firm cost function $C(y, s)$ can then
be expressed as:

\[ C(y, s) = \sum_{i=1}^{2} w_is_i + \sum_{i=1}^{2} t_id_is_i + t_0d_0q^0 + \tau \left( \alpha_0d_0q^0 + \sum_{i=2}^{1} \alpha_id_is_i \right) , \]

where \( \sum_{i=1}^{2} w_is_i \) is sourcing cost; \( \sum_{i=1}^{2} t_id_is_i \) is upstream transportation cost; \( t_0d_0q^0 \) is downstream transportation cost; and \( \tau \left( \alpha_0d_0q^0 + \sum_{i=2}^{1} \alpha_id_is_i \right) \) is transport-related carbon emissions cost based on the distance, the quantity of freight carried, and the carbon intensity of transportation mode.

Let \( T_i \equiv t_i + \tau \alpha_i \) with \( i = 0, 1, 2 \) be the total unit transportation cost, which includes a variable unit transportation cost part \( t_i \) and an environmental transportation unit cost part of carbon emissions \( \tau \alpha_i \). Therefore, establishing a carbon tax on carries means increasing the variable unit transport cost \( t_i \) with \( i = 0, 1, 2 \). In a sustainability context, fuel taxes can also be seen as a scenario of increasing transport costs to reduce carbon emission from transportation as each unit of fuel used generates a certain amount of emissions. Firm cost can be rewritten as follows:

\[ C(y, s) = T_0d_0q^0 + \sum_{i=1}^{2} (w_i + T_id_i)s_i , \]

where \( w_i + T_id_i \) represents the delivered price of a shipped input unit, composed of the the per-unit procurement and total transportation costs. The optimal economic efficient location that minimizes firm cost \( C \) is then described by

\[ C^* = \min_{y,s} C(y, s) \quad (1) \]

subject to

\[ q^0 = f(s_1, s_2) \quad (2) \]
\[ y \geq 0 \quad (3) \]
\[ s \geq 0 \quad (4) \]
Constraint (2) requires that production possibilities should be respected, which means that
the firm will use the production techniques that will allow it to meet the total demand
(technology constraint). Constraints (3) and (4) impose variable positivity. This model also
encapsulates an interesting formulation representing the total carbon footprint generated
during upstream and downstream transport phases noted as $E$ and expressed as follows:

$$E(y, s) = \alpha_0 d_0 q^0 + \sum_{i=1}^{2} \alpha_i d_i s_i$$

Transport-related carbon emissions minimizing location is then given by

$$E^* = \min_{y, s} E(y, s)$$

with respect to constraints (2), (3) and (4). The establishment of this second formulation
would enable us to compare optimal firm location circumstances according to two scenarios:
(i) The location $y^*$ that minimizes the firm’s total cost (economic efficiency), and (ii) the
location $y^e$ that minimizes transport’s carbon footprint (environmental efficiency). The
aim of this comparison is to examine whether the carbon tax can be used as an effective
instrument to induce a cost-minimizing location to arrive at an emission-minimizing outcome.
In the sequel, we first assume a Leontief production function (complementary essential in-
puts) and then we consider that inputs are combined with a Cobb-Douglas technology. For
the rest of the paper, indexes $i$ and $j$ will be equal to 1 or 2 when they are not explicit.

4. Leontief technology

The Leontief production function is characterized by a linear relationship between inputs
and outputs, a fixed proportion between inputs and a constant return to scale (Varian and
Repcheck (2010)). Then, with two inputs, to produce an output unit, it is necessary to
have $a_1$ units of input 1 and $a_2$ units of input 2. These positive parameters, called technical
coefficients, correspond to the number of each required per unit of component in the classical
notion of BOM. The latter has been specifically used in some location models under the
carbon pricing scheme as the production technology (see, e.g., Ramudhin et al. (2010),
Chaabane et al. (2012)).
4.1. Cost minimizing and emission-minimizing locations

The demand of each input $s_i$ is fixed as the production volume $q^0$ is given and the technical coefficients $a_i$ are defined by the technology so that

$$s_i = a_i q^0.$$ 

These supply quantities are independent from the input mill price $w_i$ because each input is available only at one input source (no competition between suppliers or geographical substitution into the market structure). The firm cost function $C(y)$ is then given by

$$C(y) = \left[ T_0 d_0 + \sum_{i=1}^{2} (w_i + T_i d_i) a_i \right] q^0.$$ 

Because the firm’s objective function is linear in one-dimensional space, the solutions are necessarily at the segments’ corners, then $y^* = 0$, $y^* = x$, or $y^* = z$. Therefore, it is sufficient to consider the following two cases:

- Case a: if $0 \leq y \leq x$, then $\frac{dC_a}{dy} = -(T_0 + a_1 T_1 - a_2 T_2) q^0$

- Case b: if $x \leq y \leq z$, then $\frac{dC_b}{dy} = -(T_0 - a_1 T_1 - a_2 T_2) q^0$

These results show that optimal location depends only on production technology and the relative per-unit total transport prices parameters. Consider the situation where the firm is indifferent to being located between two endpoints of the segments $[S_2, S_1]$ and $[S_1, M]^2$

- If $\frac{dC_a}{dy} = 0$, the firm is indifferent to being located at $S_2(0)$ or $S_1(x)$, and

  $$a_2 = P_{0x}(a_1) \equiv \frac{T_1}{T_2} a_1 + \frac{T_0}{T_2}$$

- Likewise, if $\frac{dC_b}{dy} = 0$, the firm is indifferent to being located at $S_1(x)$ or $M(z)$, and

  $$a_2 = P_{xz}(a_1) \equiv -\frac{T_1}{T_2} a_1 + \frac{T_0}{T_2}$$

^2Notice that when $\frac{dC_a}{dy} = 0$ or $\frac{dC_b}{dy} = 0$, the firm is not only indifferent to being located at the endpoints of the segment $[S_2, S_1]$ or $[S_1, M]$, respectively, but over all the points of each segment. Therefore, interior solutions may exist.
Thus, we can determine the optimal location conditions of each corner solution in each segment by defining the following thresholds:

- If \( a_2 < P_{0x}(a_1) \) then \( S_2(0) \) is preferred to \( S_1(x) \); that is \( y^* = 0 \).
- If \( a_2 > P_{0x}(a_1) \) then \( S_1(x) \) is preferred to \( S_2(0) \); that is \( y^* = x \).
- If \( a_2 < P_{xz}(a_1) \) then \( S_1(x) \) is preferred to \( M(z) \); that is \( y^* = x \).
- If \( a_2 > P_{xz}(a_1) \) then \( M(z) \) is preferred to \( S_1(x) \); that is \( y^* = z \).

Figure 2 is a representation of all possible firm’s cost-minimizing locations in the space of all combinations of the technical coefficients \((a_1; a_2)\). It is shared in three zones: in the north part, the firm must be located in \( S_2 \), in the east part, the firm must be located in \( S_1 \), and in the south part, the firm must be located in \( M \). The surfaces of these three zones are delimited by the thresholds of optimal location choice \( P_{0x} \) and \( P_{xz} \) on which the firm is indifferent to being located in two neighboring zones. They are completely symmetrical (same slope value for both lines \((T_i/T_j)\) but with opposite signs), \( P_{0x}(0) = P_{xz}(0) = T_0/T_i \), \( P_{0x}(a_1) > P_{xz}(a_1) \) when \( a_1 > 0 \), and when \( a_2 = 0 \) then \( a_1 = T_0/T_i \). The surface size of a zone depends on the relative per-unit total transport prices \( T_i/T_j \) with \( i = 0, 1, j = 1, 2 \) and \( i \neq j \). For instance, when the total transport unit cost of an output \( T_0 \) is much higher than the total transport
unit cost of the inputs $T_1$ and $T_2$, then the ratios $\frac{T_0}{T_1}$ and $\frac{T_0}{T_2}$ will be higher. Consequently, the surface area of the zone $y^* = 0$ will increase, and the firm is more likely to be located at the market $M$. These results and analyses are described in the next lemma.

**Lemma 1.** *The location that minimizes the cost is given by $y^* = 0$ if and only if $a_2 \geq P_{0z}(a_1)$, $y^* = x$ if and only if $P_{0x}(a_1) \geq a_2 \geq P_{xz}(a_1)$ and $y^* = z$ if and only if $a_2 \leq P_{xz}(a_1)$.*

In the same way as for cost-minimizing location, we determine the firm location $y^e$ that minimized the transport-related emissions $E(y)$ for a given level of production $q^0$ (see Appendix A.1). More precisely, we show that

**Lemma 2.** *The location minimizing emissions is given by $y^e = 0$ if and only if $a_2 \geq E_{0x}(a_1)$, $y^e = x$ if and only if $E_{0x}(a_1) \geq a_2 \geq E_{xz}(a_1)$ and $y^e = z$ if and only if $a_2 \leq E_{xz}(a_1)$.*

Such as $a_2 = E_{0x}(a_1) \equiv \frac{\alpha_1}{\alpha_2}a_1 + \frac{\alpha_0}{\alpha_2}$ occurs when the firm is indifferent to being located in $S_2(0)$ or in $S_1(x)$, and $a_2 = E_{xz}(a_1) \equiv -\frac{\alpha_1}{\alpha_2}a_1 + \frac{\alpha_0}{\alpha_2}$ occurs when the firm is indifferent to being located in $S_1(x)$ or in $M(z)$.

![Figure 3: Pollution minimizing location in $(a_1 - a_2)$ space.](image-url)
Therefore, to minimize the environmental function, the firm’s location is explained by the values of the technical coefficients of its inputs and the ratios of the unit carbon intensity of the transportation modes linked by the linear relationships $E_{0x}$ and $E_{xz}$. The conditions of the firm’s emission-minimizing locations are completely independent from the carbon tax $\tau$. Thus, an increase in $\tau$ will not directly improve the ecological outcome but it would only react through a firm location change.

Figure 2 represents all possible firm’s emission-minimizing locations in the space $(a_1; a_2)$. The surface zones ($y^e = 0$, $y^e = x$, and $y^e = z$) are delimited by the environmental thresholds $E_{0x}$ and $E_{xz}$. They are also completely symmetrical (same slope value for both lines $\alpha_{1}/\alpha_{2}$ but with opposite signs), $E_{0x}(0) = E_{xz}(0) = \frac{\alpha_{0}}{\alpha_{2}}$, $E_{0x}(a_1) > E_{xz}(a_1)$ when $a_1 > 0$, and when $a_2 = 0$ then $a_1 = \frac{\alpha_{0}}{\alpha_{1}}$. The surface size of a zone in this case depends on the ratios between carbon emissions units $\frac{\alpha_{i}}{\alpha_{j}}$ with $i = 0, 1, j = 1, 2$ and $i \neq j$.

4.2. Ecological outcome vs economic outcome: the role of carbon tax

Having identify the conditions of the firm’s cost-minimizing and emissions-minimizing locations, we can analyze when they do coincide (or deviate), under the Leontief setting. When we cross the conditions’ solutions depicted in Figures 2 and 3, we can observe that for some input combinations $(a_1; a_2)$, a firm cannot be located optimally for both functions, as shown in the hatched areas of Figure 4 (see also Appendix A.2). This can be resumed in the next proposition:

**Proposition 1.** Regardless of a carbon tax ($\tau \geq 0$), the cost-minimizing location corresponds to the location minimizing emissions if and only if one of the three following conditions holds:

- $a_2 > \max\{P_{0x}(a_1), E_{0x}(a_1)\}$
- $a_2 < \min\{P_{xz}(a_1), E_{xz}(a_1)\}$
- $\min\{P_{0x}(a_1), E_{0x}(a_1)\} > a_2 > \max\{P_{xz}(a_1), E_{xz}(a_1)\}$
The cost-minimizing and emissions-minimizing solutions do not always overlap because the per-unit monetary transportation costs of the commodities are not proportional to the corresponding per-unit emissions. As a result, the cost-minimizing location may induce excess pollution. This theoretical result is supported by some empirical studies (see, e.g., the experiments performed by You and Wang (2011) in the biomass industry and by Ramudhin et al. (2010) in the steel industry).

We consider now the impacts of changes in carbon tax at reducing the gap between the two objectives. When the ratios differ: $\frac{\alpha_1}{\alpha_2} \neq \frac{T_1}{T_2}$, $\frac{\alpha_0}{\alpha_1} \neq \frac{T_0}{T_1}$, and $\frac{\alpha_0}{\alpha_2} \neq \frac{T_0}{T_2}$, it can be easily shown that an increasing carbon tax makes cost-minimizing thresholds $P_{0x}$ and $P_{xz}$ converge to meet pollution-minimizing thresholds $E_{0x}$ and $E_{xz}$, respectively. This triggers firm that has a conflict between the economic and environmental criteria to relocate toward to a lower level of emissions location. Otherwise, for firm that does not have a conflict, a higher carbon tax cannot hurt the ecological outcome, regardless of the combination $(a_1, a_2)$, but it does increase the overall cost. In other words, a firm remains in $S_1$, $S_2$ or $M$ when the carbon tax increases. To summarize,

\[ \text{Nevertheless, it appears that when} \quad \frac{\alpha_1}{\alpha_2} = \frac{T_1}{T_2}, \quad \frac{\alpha_0}{\alpha_1} = \frac{T_0}{T_1}, \quad \text{and} \quad \frac{\alpha_0}{\alpha_2} = \frac{T_0}{T_2}, \quad \text{then all combinations} \quad (a_1, a_2) \quad \text{lead to both economic and environmental efficiency (curves in Figure 4 are confounded). In particular, in a} \]
Proposition 2. Assuming a Leontief production technology, an increase in carbon tax makes it more likely that the cost-minimizing location corresponds to the pollution-minimizing location.

5. Cobb-Douglas technology

The technology studied in this section is fundamentally different from that in the previous section. It is another polar specification of the production function \( f \) in which the inputs are imperfectly substitutable and numerous technological combinations may exist to produce goods (Varian and Repcheck (2010)). For example, feed mixture for livestock requires combining several quantities of cereals (e.g. wheat, barley, soy) to cover livestock nutritional needs. Then, the quantities of the cereals in a food recipe may be varied but to a limited extent to respect the nutritional intake (this is a classical problem of blending inputs).

5.1. Technology and Input demand

The Cobb-Douglas functional form is defined as follows in our case with two inputs:

\[
q^0 = f(s_1, s_2) = a_0 \prod_{i=1}^{2} s_i^{a_i},
\]

where \( a_0 > 0 \) is the Total-Factor Productivity (TFP). It is defined as the part of production that is not explained by the amount of inputs used in production as technical progress (Solow (1957)). Its value is exogenous and determined by the available technology. \( a_i > 0 \) is a partial production elasticity coefficient and refers to the change produced in output level \( q^0 \) due to the change in input quantity \( s_i \) while keeping input quantity \( s_j \) constant with \( j \neq i \). The value of \( a_i \) is also exogenous and is determined by the available technology. Furthermore, mono-modal transport scheme, the firm chooses the best location in both dimensions regardless of the level of carbon tax. In other words, the choice of multi-modal transport scheme may hurt the ecological outcome. Consequently, a carbon tax is not justified for firms which use only one transportation mode (supply and deliver sides). But such a result can be considered as an extreme case.
$a_1 + a_2$ is called the total production elasticity. The Cobb-Douglass production function exhibits constant returns to scale when $a_1 + a_2 = 1$ (meaning that doubling the usage of $s_1$ and $s_2$ will also double output $q^0$) and decreasing (respectively increasing) returns to scale when $a_1 + a_2 < 1$ (respectively $a_1 + a_2 > 1$).

The function $f$ can be depicted by an isoquant (see Figure 5). It represents the set of all combinations of input quantities $(s_1, s_2)$ that leads to the same level of output $q^0$. For example, the points $A$ and $B$ allow the firm to produce exactly the same level of production $q^0$ as all the other points that belong to the isoquant. The form of this isoquant is important. It is a hyperbolic curve and therefore decreasing and convex. This allows the firm to have flexibility in substituting among input quantities. Therefore, a change from point $B$ to point $A$ corresponds to a reduction on input quantity $s_1$ compensated by an increase in input quantity $s_2$ to keep the level of production $q^0$ constant.

In the latter, we will assume that $a_1 + a_2 = 1$, which implies a constant return to scale and log-linear form possibility for the Cobb-Douglass production function. This choice is justified by the fact that the Leontief production function is characterized by a constant return to scale and a linear form. Then, it would be better to assume the same properties for the Cobb-Douglass production function to isolate and emphasize the role of the input.
In Cobb-Douglas production setting, the firm not only optimizes its location but it also determines the allocation between the two inputs (input mix). Given the level of production $q^0$, the firm minimizes the upstream costs regardless of firm location $y$ under the Cobb-Douglas technological constraint, which leads to the following individual demand for each input (see Appendix B.1):

$$s_i^* = \left( \frac{q^0}{a_0} \right) \left[ \frac{a_i(w_i+T_i d_i)}{a_j(w_j+T_j d_j)} \right]^{a_j}$$

(5)

with $i \neq j$, and where the ratio $\frac{w_i+T_i d_i}{w_j+T_j d_j}$ represents the relative delivered price between the two inputs. Thus, an increase in the carbon tax (through an increase in total transport unit cost $T_i$) will alter the relative delivered price of a shipped input. Consequently, the firm will adjust its input quantity choices for each level of carbon tax $\tau$.

5.2. Cost-minimizing location and ecological outcome

Having specified the functional form of the Cobb-Douglas production function and identified the optimal set of supply quantities that are combined, we can determine the optimal firm location. Replacing $s_1^*$ and $s_2^*$ (5) in firm cost leads to:

$$C(y) = \nu(y)q^0,$$

with

$$\nu(y) \equiv T_0d_0 + \Psi \prod_{i=1}^2 (\zeta_i d_i + 1)^{a_i}, \quad \Psi \equiv \frac{1}{a_0} \prod_{i=1}^2 w_i^{a_i} \sum_{i \neq j} \left( \frac{a_i}{a_j} \right)^{a_j}, \quad \text{and} \quad \zeta_i \equiv \frac{T_i}{w_i},$$

where $\nu(y)$ is the marginal cost of production related to the location $y$, $\Psi$ is a bundle of exogenous parameters that are independent from distances and $\zeta_i$ is the ratio of the total transport unit cost to the purchasing unit cost.

The cost function $C(y)$ is concave over the intervals $[0, x]$ and $[x, z]$ as $\frac{d^2 C}{dy^2} < 0$ (see Appendix B.2). Hence, there are three candidates for the cost-minimizing location: $y^* = 0$, ...
$y^* = x$, and $y^* = z$. The firm cost associated with each optimal location candidate is expressed as follows:

- $C_0 = \nu_0 q^0 = [T_0 z + \Psi (\zeta_1 x + 1)^{a_1}] q^0$,
- $C_x = \nu_x q^0 = [T_0 (z - x) + \Psi (\zeta_2 x + 1)^{a_2}] q^0$,
- $C_z = \nu_z q^0 = [\Psi (\zeta_1 (z - x) + 1)^{a_1} (\zeta_2 z + 1)^{a_2}] q^0$.

The cost-minimizing locations are not necessarily those that minimize the total transportation costs as for Leontief technology because a compromise emerges between the transportation and production costs (see Appendix B.3).

Furthermore, the ecological outcome is given by:

$$E(y) = \theta(y) q^0$$

with

$$\theta(y) \equiv \alpha_0 d_0 + \Psi \prod_{i=1}^{2} (\zeta_i d_i + 1)^{a_i} \left( \sum_{i=1}^{2} \frac{\alpha_i d_i}{T_i d_i + w_i} \right),$$

where $\theta(y)$ is the marginal emission of production related to the location $y^*$.

The environmental function $E$ shows an interesting feature. Unlike the Leontief setting, where the emissions level were completely independent from $\tau$, the carbon tax affects directly the level of emissions through the input quantities ($s_1^*, s_2^*$) under a Cobb-Douglas substitution structure. Although the tax is identical for both inputs, it changes the relative delivered price of a shipped input and therefore its demand quantity which changes the level of emissions. Because it is difficult to explore analytically the environmental function in order to determine pollution minimizing location, we associated to each cost-minimizing location candidate:

---

4The minimization of the cost function $C$ (resp. environmental function $E$) returns to minimize the marginal cost $\nu$ (resp. marginal emissions $\theta$) of production of an output unit at the location $y$. This characteristic of the cost function has already been proved for Production-Location problems when the production function is homogeneous under the name of ”separability theorem ” and whose statement is as follows: ”the location which minimizes the total cost for a unit of output must minimize the total cost for all levels of production” (Hurter and Venta (1982)).
$y^* = 0$, $y^* = x$, and $y^* = z$ the following level of pollution, respectively:

\[
E_0 = \theta_0 q^0 = \left[ \alpha_0 z + \Psi \frac{\alpha_1 x}{T_1 x + w_1} (\zeta_1 x + 1)^{a_1} \right] q^0,
\]

\[
E_x = \theta_x q^0 = \left[ \alpha_0 (z - x) + \Psi \frac{\alpha_2 x}{T_2 x + w_2} (\zeta_2 x + 1)^{a_2} \right] q^0,
\]

\[
E_z = \theta_z q^0 = \left[ \Psi \left( \frac{\alpha_1 (z - x)}{T_1 (z - x) + w_1} + \frac{\alpha_2 z}{T_2 z + w_2} \right) (\zeta_1 (z - x) + 1)^{a_1} (\zeta_2 z + 1)^{a_2} \right] q^0.
\]

Thereafter, we perform some simulations to determine whether the cost-minimizing location matches at least the lowest pollution level observed among these three candidates. Several numerical examples reveal that the cost-minimizing solutions would not always coincide with the lowest level of emissions because the per-unit procurement and monetary transportation costs of the commodities are not proportional to the corresponding per-unit emissions. Figure 6 represents the results of one of these situations. We have considered $q^0 = 100$, $a_0 = 1$, $a_1 = 0.5$, $a_2 = 0.5$, and $\tau = 10$. The per-unit procurement and monetary transportation costs; and the per-unit emissions values of the commodities are reported in Table 1. The left side of Figure 6 shows the firm costs at the locations $y = 0$, $y = x$, and $y = z$, regardless of the position $x$ of the supplier $S_1$ on the normalized segment $[S_2(0), M(1)]$ (i.e. $0 \leq x \leq 1$). Cost-minimizing location is at supplier $S_1$ regardless of its location. The right side of Figure 6 depicts the associated pollution levels. It indicates that the lowest observed level of pollution is at supplier $S_2$ regardless of the location of supplier $S_1$. Thus, the location that minimizes cost does not correspond to the lowest level of pollution among the three stakeholder locations, and therefore also to the pollution-minimizing location.

<table>
<thead>
<tr>
<th>Parameter</th>
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<th>$w_2$</th>
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<th>$t_1$</th>
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We consider now the impacts of changes in carbon tax. To disentangle the various effects at work, it is both relevant and convenient to distinguish between two cases: first, a short-run equilibrium, in which firm location is assumed to be fixed (i.e. $y$ is exogenous). In this
case, the location decision is already made, and adjustments can only be performed on input quantity choices in response to a change in carbon tax (effect on tactical decision). Second, a long-run equilibrium, in which a firm is free to relocate (i.e. \( y \) is endogenous). Then, the firm can adjust both its location and input quantity choices in response to a change in carbon tax (effect on both strategic and tactical decisions).

### 5.3. Short-run analysis

Facility location \( y \) is given, and the impact of the carbon tax on transport-related emissions can be written as follows:

\[
\frac{dE}{d\tau} = \alpha_1 d_1 \frac{ds_1^*}{d\tau} + \alpha_2 d_2 \frac{ds_2^*}{d\tau} = -\frac{1}{w_2 + T_2d_2} s_2^* a_1 \chi^2 < 0
\]

with

\[
\chi \equiv \frac{\alpha_2 d_2(w_1 + T_1d_1) - \alpha_1 d_1(w_2 + T_2d_2)}{(w_1 + T_1d_1)(w_2 + T_2d_2)}.
\]

Thus, the level of transport-related emissions shrinks with a higher carbon tax regardless of the combination choice of the input quantities \((s_1, s_2)\). Let examine the adjustment mechanism of the input quantities in response to a change in carbon tax rate. From firm’s viewpoint, the production combinations \((s_1, s_2)\) correspond to the upstream cost noted \( c \):

\[
c = (w_1 + T_1d_1)s_1 + (w_2 + T_2d_2)s_2
\]
that can be written as

\[ s_2 = \frac{-w_1 + T_1 d_1}{w_2 + T_2 d_2} s_1 + \frac{c}{w_2 + T_2 d_2}. \]

In the space \((s_1, s_2)\), that equation defines an *isocost* line of slope

\( \frac{-w_1 + T_1 d_1}{w_2 + T_2 d_2} \)

that corresponds to the relative delivered price between the two inputs. When we change the value of \(c\), we obtain a set of isocost lines. All the points of an isocost line correspond to the same cost \(c\), and higher isocost lines are associated with higher costs. Conversely, the firm must choose the set of input quantities that allows it to produce exactly the output level \(q^0\) that corresponds to the isoquant (see Figure 7) of the slope

\[ \frac{ds_2}{ds_1} \]

that measures the rate at which the firm must substitute one input for the other while keeping the quantity of output \(q^0\) constant. This rate is called the *Marginal Rate of Substitution (MRS)*. It measures the trade-off between the two input quantities at the production level. Minimizing the cost \(c\) can therefore be expressed as follows: this is to find the point \(A\) on the isoquant curve associated with the lowest possible isocost line. Such a point \(A\) is represented in Figure 7. Point \(A\) is characterized by a tangency condition: the slope of the isoquant curve must be equal to the slope of the isocost line. In other words, the MRS must be equal to the relative delivered price between the two inputs:

\[ \frac{-ds_2}{ds_1} = \frac{-w_1 + T_1 d_1}{w_2 + T_2 d_2} \]

In the same way, but from an ecological standpoint, the input combinations \((s_1, s_2)\) correspond to the carbon emissions level \(e\):

\[ e = (\alpha_1 d_1) s_1 + (\alpha_2 d_2) s_2 \]

which can be written as

\[ s_2 = \frac{-\alpha_1 d_1}{\alpha_2 d_2} s_1 + \frac{e}{\alpha_2 d_2}. \]
In the space $(s_1, s_2)$, that equation defines an *isoemissions* line of the slope

$$-\frac{\alpha_1 d_1}{\alpha_2 d_2}$$

that corresponds to the ratio of carbon emissions between the two inputs. When we change the value of $e$, we obtain a set of isoemissions lines. Minimizing carbon emissions from the supply side returns to find the point $B$ on the isoquant curve associated with the lowest possible isoemissions line. Such a point $B$ is represented in Figure 7 and characterized by a tangency condition that is:

$$-\frac{ds_2}{ds_1} = -\frac{\alpha_1 d_1}{\alpha_2 d_2}$$

The point $A$ (respectively $B$) corresponds to the choice of input quantities $(s^*_1, s^*_2)$ (resp. $(s^e_1, s^e_2)$) that minimizes the upstream cost function $c$ (resp. the upstream carbon emissions function $e$) (see Appendix B.1 and Appendix B.4). Standard calculation shows that an increasing carbon tax makes more likely the optimal input quantities $(s^*_1, s^*_2)$ converge toward input quantities $(s^e_1, s^e_2)$ that generates the lowest emission that can be emitted. Thus, even when input substitution is possible, a larger weight on emissions (due to a higher carbon tax) in the effective shipping cost shifts the input proportions in the direction that lowers emissions. To sum up,
Proposition 3. For a given firm location, when the firm has substitution opportunities among input quantities, an increased carbon tax reduces transport-related carbon emissions as the relative delivered prices of inputs converge to the system of relative prices, inducing the lowest level of carbon emissions.

Following this result, we can expect that an increase in the carbon tax would increase the demand for the less polluting input (i.e. the input exhibiting the lowest level of transport-related emissions per unit of shipped input $\alpha_i d_i$) at the expense of the more polluting input. In other words, the firm would purchase more input quantities for which it would pay a lower carbon tax when the latter increases. However, as shown in Appendix B.5, even though the total emissions decline when the carbon tax increases, the demand for the more polluting input may increase under some conditions. More precisely, we show that

Lemma 3. For a given location and a given level of production, the quantity of the more polluting input (per unit of shipped input $\alpha_i d_i$) increases when the carbon tax increases if and only if the firm location is such that $\frac{\alpha_2}{\alpha_1} d_2 < d_1 < \hat{d}_1$ when $S_1$ is more polluting or $\hat{d}_1 < d_1 < \frac{\alpha_2}{\alpha_1} d_2$ when $S_2$ is more polluting with

$$\hat{d}_1 \equiv \frac{\alpha_2 w_1 d_2}{\alpha_1 w_2 + d_2 (\alpha_1 t_2 - \alpha_2 t_1)} > 0.$$

Therefore, by improving its overall cost through an adjustment of its input quantities, the firm will always reduce its carbon emissions level. However, when the amount of the more polluting input increases, the carbon emission mitigation will be relatively lower than if the amount of the less polluting input was increased. In this latter configuration, the carbon emissions reduction would be more significant. The circumstances of Lemma 3 have been refined with regard to the transport modes that the firm uses for its input supply (see Appendix B.6)\[5\]

5.4. Long-run analysis

In this part, we investigate how both firm’s location and production decisions are affected in response to a change in the carbon tax rate in a long term perspective. Based on numerical

\[5\]Note that the results for the short-run analysis are obtained by assuming firm interior locations.
illustrations, we found that increasing carbon taxation may trigger the firm to change its optimal facility location and inputs mix in order to re-optimize its overall cost. Such a relocation decision may coincide with a significant ecological improvement (*a downward jump of the emissions level*) or may worsen the ecological outcome (*an upward jump of the emissions level*). In the sequel, we propose two explicit numerical parameter settings where emissions could change in either direction as the optimal location changes.

**A downward jump of emissions level**

We built our example by normalizing the line segment \([S_2, M]\) to 1, and we considered the relative location of supplier \(S_1\) in this segment \(x\) equal to 0.2. Let the demand level \(q^0 = 100\), and the technical production parameters \(a_0 = 1\), \(a_1 = 0.3\), and \(a_2 = 0.7\). The per-unit procurement and monetary transportation costs; and the per-unit emissions values of the commodities are reported in Table 2.

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<th>Parameter</th>
<th>(w_1)</th>
<th>(w_2)</th>
<th>(t_0)</th>
<th>(t_1)</th>
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Table 2: Data for the example of a downward jump of emissions level

Figure 8 provides schematic sketches of the firm’s optimal total cost and the resulting GHG emissions.

Figure 8: A double performance relocation.

Figure provides schematic sketches of the firm’s optimal total cost and the resulting GHG emissions.
emissions graphs as the carbon tax evolves \((\tau \in [0,30])\). When \(\tau < \bar{\tau} = 6.788\) (\(\bar{\tau}\) is the critical carbon price which causes firm relocation decision), it is more economically efficient for the firm to locate at the supplier \(S_2\); to carry the input 1 from the supplier \(S_1\); and to deliver the output to the market \(M\) (see the left side of Figure 8). As already shown in Appendix B.3, \(S_2\) is preferred to \(S_1\) and \(M\) is more likely to occur when \(T_0\) is sufficiently low, \(\zeta_2\) is sufficiently high, and \(x\) is close to 0. These conditions are fulfilled in this example. In fact, supplier \(S_1\) is located relatively near to supplier \(S_2\) \((x = 0.2)\). In addition, when the carbon tax is relatively low, output transport \(T_0 = 2 + 0.4\tau\) is sufficiently low and the ratio of the total transport unit cost to the purchasing unit cost of input 2 \(\zeta_2 = 4 + 0.2\tau\) is sufficiently high while the ratio of the total transport unit cost to the purchasing unit cost of input 1 \(\zeta_1 = 1 + 0.3\tau\) is sufficiently low (see Figure 9). Although the total firm cost increases at the supplier \(S_2\), carbon emissions shrinks with a higher carbon price (see the right side of Figure 8) because the firm gradually substitutes the amount quantity of the input 1 (the more polluting input per unit of shipped input) at the expense of the amount quantity of the local input 2 (the less polluting input) for which it would pay a lower carbon tax.

![Figure 9: Effective shipping costs of commodities according to carbon price evolution.](image)

When \(\tau \geq \bar{\tau} = 6.788\), the firm jumps to supplier \(S_1\) to minimize its total cost (see the left side of Figure 8). This may be explained by the fact that when carbon tax is relatively high,
the firm would have the incentive to reduce the relatively high increases costs effect in $T_0$ and $\zeta_1$ compared to $\zeta_2$ (see Figure 9) by moving closer to the output market (i.e., supplier $S_1$). Such a relocation has been accompanied by a new substitution among the input quantities and, thus, new shipping patterns. The firm purchases a substantial amount of the input 1 obtained at a relatively low delivered price, and reduces the use of the input 2. These new shipping patterns cause a discontinuity and a downward jump of carbon emissions (see the right side of Figure 8). The total rate of reduction of emissions at the critical price triggering the firm relocation is estimated at around $20.86\%$. Furthermore, the emission reduction rate after the critical price continues to decline as the optimal amount of input 2 (the more polluting input per unit of shipped input in this case) decreases gradually due to a higher carbon tax.

Hence, since the firm can partially influence the input delivered price by adjusting its location, it is more economically beneficial to relocate the production plant towards the increasingly expensive source of input and use more of it. This also allows to a better curb of transport carbon emissions.

**An upward jump in emissions level**

As in the previous case, we built our example by normalizing the line segment $[S_2, M]$ to 1, and we considered the relative location of supplier $S_1$ in this segment $x$ equal to 0.9. The demand level $q_0 = 100$, and the production parameters $a_0 = 2$, $a_1 = 0.55$, $a_2 = 0.45$ and the per-unit procurement and monetary transportation costs; and the per-unit emissions values of the commodities are reported in Table 3.

<table>
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Figure 10 is similar to Figure 8 in that it shows how an increase in the carbon tax ($\tau \in [0, 30]$) affects the firm location choice and carbon emission amount. When $\tau < \tau_1 = 8.157$ ($\tau_1$ is the critical carbon price which causes firm relocation decision), firm optimal location is at the
Figure 10: An increase in carbon tax does not coincide with an ecological improvement.

supplier $S_2$ since the effective shipping cost of input 2 $\zeta_2$ is relatively more expensive than output transport $T_0$ and the effective shipping cost of input 1 $\zeta_1$ (see Figure 11) so that it pays to locate at the input 2 source and to ship input 1 and deliver output. The same effects holds for transport-related carbon emissions mitigation.

Figure 11: The effective shipping costs of commodities according to carbon price evolution.

When $\tau_1 = 8.157$, firm changes its optimal location to supplier $S_1$ and its optimal input mix in order to re-optimize its total cost. Such a relocation induces a discontinuity and an upward
jump of carbon emissions (see the right side of Figure 10). The total emissions increased by 35.9% compared with that before the jump. After the upward jump, carbon emissions decline with a higher carbon tax (since the firm continues to reduce the proportion of input 2) until they became less than before the upward jump when \( \tau > \tau_2 = 25.683 \). Therefore, even though the firm location and production adjustments react to marginal change in carbon taxation, it does not coincide with an ecological improvement (as long as the rise in carbon tax rate is not too large).

**Basic intuitions**

To better understand our results, we examine the change in the combination of input quantities along the isoquant curve before and after a firm relocation decision in the previous two examples (location change from supplier 2 to supplier 1). Before the firm decides to change its optimal facility to supplier \( S_1 \), the input mix that minimizes the upstream cost function \( c \) (resp. upstream carbon emissions function \( e \)) corresponds to the point \( A \) (respectively \( B \)) (see Figure 12). When the carbon tax rate gradually increases, point \( A \) converges toward point \( B \) (the optimal amount of input 1 decreases gradually) as the relative delivered price of input 2 converges to the relative delivered price leading to the lowest level of carbon emissions (same mechanism as in the short-term case where the location is given (see Figure 7)). However, when carbon tax rate crosses above some threshold, this convergence stops as the firm relocates its facility to supplier 1. Thus, the optimal input mix changes dramatically as the relative delivered prices between inputs change. The new input mix is represented in the isoquant curve by the point \( A' \). This latter leads to new shipping patterns that can generate either a lower level of carbon emissions (a **downward jump of the emissions level**) or a higher level of carbon emissions (an **upward jump of the emissions level**). Nevertheless, a larger weight on emissions (due to a higher carbon tax) in the effective shipping costs shifts the new input proportions \( A' \) in the direction that lowers emissions at point \( B' \) (the optimal amount of input 2 decreases gradually).

These results illustrate how a marginal change in carbon tax can cause dramatic changes in
firm optimal location, the combination of inputs, and the ecological outcome. Accounting for these effects makes the impact of a higher carbon tax more ambiguous when the input materials are substitutable. Mainly, because their net effects depend on whether the new spatial pattern is better or worse from an environmental viewpoint. These results also provide some policy insights to design an efficient carbon tax on transport-related carbon emissions. According to the initial of carbon taxation, the increase in carbon tax should not occur gradually or marginally in some configurations. Rather, there should be a very strong increase so that it leads to location adjusting without an upwards jumps in emissions levels.

6. Conclusion

This paper studies the effect of carbon taxation on firms facility location decisions. Based on a stylized facility location model on a linear network, we evaluate the cost-minimizing and emissions-minimizing facility location decisions and analyze when they do/do not coincide, under two alternative production technology settings, namely Leontief and Cobb-Douglas. We conclude that a carbon tax is effective in reducing transport-related emissions in the Leontief case where input proportions are fixed; however, a carbon tax could possibly worsen ecological outcome when input proportions can be varied under a Cobb-Douglas
substitution structure. Therefore, we highlight the possibility of backfiring of the carbon tax. Taxes are known to produce unintended consequences, and this appears to be one such instance especially because multiple variables (input mix and location) are allowed to vary simultaneously.

Our paper addresses a major issue. When assessing the merits of carbon taxation policies, the existing literature disregards one major problem: a higher carbon tax changes the combination of inputs. Thus, our finding implies that the studies of carbon taxation policy should be also conducted within a framework in which production technology can allow substitution among inputs. We expect that our paper touches on an important research topic and sheds new light on facility location analysis when we study the efficiency of environmental policies.

Our approach, however, has limitations that can provide guidance for further research and warrants in-depth studies in the field. The key causal assumption in the model is that firms willingly move locations around in response to changes in carbon tax on transportation. Facility location decisions are typically made during very different stages/phases in the firm’s decision horizon involving such considerations as regional labor cost/availability, and tax benefits that need to be addressed in a comprehensive and integrated model. In the same vein, the model only considers emissions from transportation but not from production. As the driver of the main result is the possibility of input substitution, it might be interesting to explore the emission implications due to changes of BOM by including the production-related emissions in the analysis.

We recognize that is a rather strong assumption to consider a one-dimensional linear network topology to represent the driving forces of facility location problems in general. Therefore, the generalization of the extended ”PLP” model through two-dimensional space and several stakeholders (multivendor, multi-market, and multi-location) under a more general setting (such as the Constant Elasticity of Substitution (CES) that has several special cases including Cobb-Douglas and Leontief functions) should be developed. There are, however, more specific applications that might fit the unit line assumption. For example, the problem of deciding the assembly location of a China-USA supply chain could resemble such a linear
Another direction to explore would be to expand the boundaries of the model. For examples, uncertainty in demand or supply; competition either for the inputs or the outputs, etc. Although it would be difficult to formally endogenize many of these complexities, but incorporating some of these factors may lead to a more interesting setting, with potentially richer insights.

Finally, from a methodological point of view, the emission-minimizing objective may not be a natural benchmark since it disregards the economic benefits of the production and the transportation. Thus, a more balanced and a well-accepted objective function such as social welfare maximization objective should be considered.

Our work is quite preliminary and it should be considered as a first step to illustrate the importance of production technology on the implications of environmental mechanisms with an economic signal on location decisions.
Appendix A. Appendix Leontief case

Appendix A.1. Pollution minimizing location

Under Leontief technology input demand, the firm transport-related emissions function $E(y)$ is given by

$$E(y) = \left[ \alpha_0 d_0 + \sum_{i=1}^{2} \alpha_i d_i a_i \right] q^0$$

with:

$$d_1 = |x - y| \iff \begin{cases} \ x - y, \text{ when F is between } S_2 \text{ and } S_1 \text{ Case a: } E^a \\ \ y - x, \text{ when F is between } S_1 \text{ and } M \text{ Case b: } E^b \end{cases}$$

Because $\frac{d^2E}{dy^2} = 0$, there is no interior solution, and the solutions are necessarily at the segments’ corners, so $y^e = 0$, $y^e = x$, or $y^e = z$ are purely ecological equilibrium solutions. This result comes from the shape of the function $E$, which is linear in $y$; thus, the derivative is constant and either positive or negative. The derivative’s sign leads to the optimal ecological location of the firm. Therefore, it is sufficient to consider the two cases:

- Case a: if $0 \leq y \leq x$, then:
  $$\frac{dE^a}{dy} = (-\alpha_0 - a_1 \alpha_1 + a_2 \alpha_2)q^0$$

- Case b: if $x \leq y \leq z$, then:
  $$\frac{dE^b}{dy} = (-\alpha_0 + a_1 \alpha_1 + a_2 \alpha_2)q^0$$

Thus, the optimal environmental location depends only on production technology and emission parameters. Consider the situation of indifference between two endpoints of the segments $[S_2, S_1]$ and $[S_1, M]$, similar to the situation of identifying the maximum profit location.

- If $\frac{dE^a}{dy} = 0$, the firm is indifferent to being located in $S_2(0)$ or $S_1(x)$, and
  $$a_2 = E_0x(a_1) \equiv \frac{\alpha_1}{\alpha_2} + \frac{\alpha_0}{\alpha_2}.$$

- If $\frac{dE^b}{dy} = 0$, the firm is indifferent to being located in $S_1(x)$ or $M(z)$, and
  $$a_2 = E_xz(a_1) \equiv -a_1 \frac{\alpha_1}{\alpha_2} + \frac{\alpha_0}{\alpha_2}.$$

Thus, we can determine the optimal location conditions from a purely environmental standpoint of each corner solution in each segment by defining the following thresholds.
- If \( a_2 < E_{xz}(a_1) \) then \( S_2(0) \) is preferred to \( S_1(x) \); that is \( y^* = 0 \).
- If \( a_2 > E_{xz}(a_1) \) then \( S_1(x) \) is preferred to \( S_2(0) \); that is \( y^* = x \).
- If \( a_2 < E_{xz}(a_1) \) then \( S_1(x) \) is preferred to \( M(z) \); that is \( y^* = x \).
- If \( a_2 > E_{xz}(a_1) \) then \( M(z) \) is preferred to \( S_1(x) \); that is \( y^* = z \).

Appendix A.2. Superposition possibilities of economic and environmental thresholds

Remember that from a firm’s standpoint, \( a_2 = P_{0x}(a_1) \) occurs when the firm is indifferent to being located in \( 0 \) or in \( x \) and, \( a_2 = P_{xz}(a_1) \) occurs when the firm is indifferent to being located in \( x \) or in \( z \). Equally, from a purely ecological view, \( a_2 = E_{0x}(a_1) \) occurs when the firm is indifferent to being located in \( 0 \) or in \( x \), and \( a_2 = E_{xz}(a_1) \) occurs when the firm is indifferent to being located in \( x \) or in \( z \). In addition, \( P_{0x}(0) = P_{xz}(0) = \frac{T_0}{T_2} \) and \( E_{0x}(0) = E_{xz}(0) = \frac{a_0}{a_2} \) whereas \( P_{xz}(\frac{T_0}{T_1}) = 0 \) and \( E_{xz}(\frac{a_0}{a_1}) = 0 \), as illustrated in Figure 5.

We have \( P_{0x}(0) \geq E_{0x}(0) \) if and only if \( t_2a_0 - t_0a_2 \leq 0 \) and \( P_{0x}(a_1) \geq E_{0x}(a_1) \). Then, we must consider two sub-cases:

- when \( t_1a_2 - t_2a_1 > 0 \), then \( a_1 \geq \hat{a}_1 \)
- when \( t_1a_2 - t_2a_1 < 0 \), then \( a_1 \leq \hat{a}_1 \)

with:

\[
\hat{a}_1 = \frac{t_2a_0 - t_0a_2}{t_1a_2 - t_2a_1}.
\]

Four configurations must be taken into account where \( P_{0x}(a_1) \geq E_{0x}(a_1) \).

1. \( t_1a_2 - t_2a_1 > 0 \) and \( t_2a_0 - t_0a_2 \leq 0 \)
2. \( t_1a_2 - t_2a_1 > 0 \) and \( t_2a_0 - t_0a_2 > 0 \)
3. \( t_1a_2 - t_2a_1 < 0 \) and \( t_2a_0 - t_0a_2 \leq 0 \)
4. \( t_1a_2 - t_2a_1 < 0 \) and \( t_2a_0 - t_0a_2 > 0 \)

The same analysis can be performed for the condition \( P_{xz}(a_1) \leq E_{xz}(a_1) \)
Appendix B. Appendix Cobb-Douglas case

Appendix B.1. Input demand

We want to determine the optimal input supply \( s^*(s_1^*, s_2^*) \) that minimizes the total cost function \( C(s, y) \) under the Cobb Douglass production function constraint, which can be written as:

\[
s^*(y) = \min_{s_1, s_2} \sum_{i=1}^{2} (w_i + t_i d_i + \tau \alpha_i d_i) s_i
\]

subject to

\[
q^0 = a_0 s_1^{a_1} s_2^{a_2} \quad \quad s \geq 0
\]

Using the functional form of the Cobb Douglass production function \( q^0 = a_0 s_1^{a_1} s_2^{a_2} \), the supply for each input can be written as follows:

\[
\begin{cases}
    s_1 = \left( \frac{q^0}{a_0} \right)^{\frac{1}{a_1}} s_2^{\frac{a_2}{a_1}} \\
    s_2 = \left( \frac{q^0}{a_0} \right)^{\frac{1}{a_2}} s_1^{\frac{a_1}{a_2}}
\end{cases}
\]  

(B.1)

The total cost function can be then rewritten according to only one of each input supply (B.1). Then, the stationary point is determined by solving the following system:

\[
\begin{cases}
    \frac{\partial C(s)}{\partial s_1} = -\frac{a_1}{a_2} (w_2 + T_2 d_2) \left( \frac{q^0}{a_0} \right)^{\frac{1}{a_2}} s_1^{-\frac{1}{a_2}} + w_1 + T_1 d_1 = 0 \\
    \frac{\partial C(s)}{\partial s_2} = -\frac{a_2}{a_1} (w_1 + T_1 d_1) \left( \frac{q^0}{a_0} \right)^{\frac{1}{a_1}} s_2^{-\frac{1}{a_1}} + w_2 + T_2 d_2 = 0
\end{cases}
\]

We therefore determine the following stationary point:

\[
\begin{cases}
    s_1 = \left( \frac{q^0}{a_0} \right) \left[ \frac{a_1(w_2 + T_2 d_2)}{a_2(w_1 + T_1 d_1)} \right]^{a_2} \\
    s_2 = \left( \frac{q^0}{a_0} \right) \left[ \frac{a_2(w_1 + T_1 d_1)}{a_1(w_2 + T_2 d_2)} \right]^{a_1}
\end{cases}
\]
The calculation of the second partial derivative shows that this stationary point is a minimum.

\[
\begin{align*}
\frac{\partial^2 C(s)}{\partial s_1^2} &= \frac{a_1}{a_2} \left( w_2 + T_2 d_2 \right) \left( \frac{q_0}{a_0} \right) \frac{1}{s_1^2} s_1^{a_2 - 2} > 0 \\
\frac{\partial^2 C(s)}{\partial s_2^2} &= \frac{a_2}{a_1} \left( w_1 + T_1 d_1 \right) \left( \frac{q_0}{a_0} \right) \frac{1}{s_2} s_2^{a_1 - 2} > 0
\end{align*}
\]

Appendix B.2. Cost-minimizing location

We show that there is no interior solution. Some standard calculations lead to

\[
\frac{dC(y)}{dy} = -T_0 q_0 + \Psi q_0 \left( \zeta_1 d_1 + 1 \right)^{a_1} \left( \zeta_2 d_2 + 1 \right)^{a_2} \left( \frac{\zeta_1}{\zeta_1 d_1 + 1} + \frac{\zeta_2}{\zeta_2 d_2 + 1} \right)
\]

If \( y > x \) then \( \frac{dd}{dy} = -1 \) and

\[
\frac{d^2C(y)}{dy^2} = -\Psi a_1 a_2 q_0 \left( \zeta_1 d_1 + 1 \right)^{a_1} \left( \zeta_2 d_2 + 1 \right)^{a_2} \left( \frac{\zeta_1}{\zeta_1 d_1 + 1} + \frac{\zeta_2}{\zeta_2 d_2 + 1} \right)^2 < 0
\]

If \( y < x \) then \( \frac{dd}{dy} = 1 \) and:

\[
\frac{d^2C(y)}{dy^2} = -\Psi a_1 a_2 q_0 \left( \zeta_1 d_1 + 1 \right)^{a_1} \left( \zeta_2 d_2 + 1 \right)^{a_2} \left( \frac{\zeta_1}{\zeta_1 d_1 + 1} - \frac{\zeta_2}{\zeta_2 d_2 + 1} \right)^2 < 0
\]

The cost function is concave over the interval \([0, x]\) and the interval \([x, z]\). Then, the cost-minimizing location is given by 0, \( x \) or \( z \).

Appendix B.3. Cost-minimizing location circumstances

Set the following differential equations:

\[
\begin{align*}
\Delta_{0x} &\equiv C_0 - C_x = \left[ T_0 x + \Psi \left( (\zeta_1 x + 1)^{a_1} - (\zeta_2 x + 1)^{a_2} \right) \right] q_0, \\
\Delta_{zx} &\equiv C_z - C_x = \left[ -T_0 (z - x) - \Psi \left[ (\zeta_1 (z - x) + 1)^{a_1} (\zeta_2 z + 1)^{a_2} - (\zeta_2 x + 1)^{a_2} \right] \right] q_0, \\
\Delta_{z0} &\equiv C_z - C_0 = \left[ -T_0 z + \Psi \left[ (\zeta_1 (z - x) + 1)^{a_1} (\zeta_2 z + 1)^{a_2} - (\zeta_1 x + 1)^{a_1} \right] \right] q_0.
\end{align*}
\]
Hence, for a given optimal solution candidate, the analysis of the signs of two differential equations in which this candidate is considered allows us to deduce sufficient and necessary conditions for the optimality of this solution. Thus, cost-minimizing location is given by $y^* = 0$ if and only if $\Delta_{0x} < 0$ and $\Delta_{x0} > 0$, $y^* = x$ if and only if $\Delta_{0x} > 0$ and $\Delta_{xx} > 0$ and $y^* = z$ if and only if $\Delta_{xz} < 0$ and $\Delta_{z0} < 0$. However, it is difficult to explore the analytical direction to determine the circumstances for each optimal location candidate. Using some numerical examples and standard calculations, we can deduce the most likely values of parameters for which the previous conditions are met. We focus here on the role of the effective shipping costs ($\zeta_1$, $\zeta_2$, and $T_0$) and the location of supplier $S_1 (x)$. We have considered $q^0 = 10$ and the following technical production parameters: $a_0 = 1$, $a_1 = 0.5$, and $a_2 = 0.5$. The results indicated that

- $S_2$ is preferred to $S_1$ and $M$ is more likely to occur when $T_0$ is sufficiently low, $\zeta_2$ is sufficiently high, and $x$ is close to $0$.

- $M$ is preferred to $S_1$ and $S_2$ is more likely to occur when $T_0$ is sufficiently high, $\zeta_2$ is sufficiently low, and $x$ is close to $z$.

- $S_1$ is more likely to be preferred when $\zeta_2$, $T_0$, and $x$ achieve relatively intermediate values.

**Appendix B.4. Minimizing pollution inputs demand**

We want to determine the optimal input supply $(s^e_1, s^e_2)$ that minimizes the total emissions level $E(s, y)$ under the Cobb Douglass production function constraint, which can be written as:

$$s^e(y) = \min_{s_1, s_2} \sum_{i=1}^{2} \alpha_i d_i s_i$$

subject to

$$q^0 = a_0 s_1^{a_1} s_2^{a_2}$$

$$s \geq 0$$

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The total carbon emissions function can be then rewritten according to only one of each input supply (see Equation (B.1)). Then, the stationary point is determined by solving the following system:

\[
\begin{align*}
\frac{\partial E(s)}{\partial s_1} &= -\frac{a_1}{a_2}(\alpha_2 d_2) \left(\frac{q^0}{a_0}\right)^{\frac{1}{a_2}} s_1^{-\frac{1}{a_2}} \alpha_1 d_1 = 0 \\
\frac{\partial E(s)}{\partial s_2} &= -\frac{a_2}{a_1}(\alpha_1 d_1) \left(\frac{q^0}{a_0}\right)^{\frac{1}{a_1}} s_2^{-\frac{1}{a_1}} + \alpha_2 d_2 = 0
\end{align*}
\]

We therefore determine the following stationary point:

\[
\begin{align*}
s_1^e &= \left(\frac{q^0}{a_0}\right) \left(\frac{a_1 d_2 \alpha_2}{a_2 d_1 \alpha_1}\right)^{a_2} \\
s_2^e &= \left(\frac{q^0}{a_0}\right) \left(\frac{a_2 d_1 \alpha_1}{a_1 d_2 \alpha_2}\right)^{a_1}
\end{align*}
\]

The calculation of the second partial derivative shows that this stationary point is a minimum.

\[
\begin{align*}
\frac{\partial^2 E(s)}{\partial s_1^2} &= \frac{a_1}{a_2} (\alpha_2 d_2) \left(\frac{q^0}{a_0}\right)^{\frac{1}{a_2}} s_1^{-\frac{a_2 - 2}{a_2}} > 0 \\
\frac{\partial^2 E(s)}{\partial s_2^2} &= \frac{a_2}{a_1} (\alpha_1 d_1) \left(\frac{q^0}{a_0}\right)^{\frac{1}{a_1}} s_2^{-\frac{a_1 - 2}{a_1}} > 0
\end{align*}
\]

Note that pollution minimizing input quantities \((s_1^e, s_2^e)\) do not depend on the tax level \(\tau\), unlike the cost minimizing input quantities \((s_1^*, s_2^*)\).

**Appendix B.5. Proof of LEMMA 3**

The pollution level depends on the distances to suppliers \((d_1 = |x - y| \text{ and } d_2 = y)\), the transport mode and the level of demand for each input. Optimal economic quantities \((s_1^*, s_2^*)\) show that the carbon tax modifies the demand for each input through an increase in its total delivered price \(w_i + T_i d_i\) but also through a change in the relative delivered price \(\frac{w_i + T_i d_i}{w_j + T_j d_j}\) with \(i \neq j\) between inputs, which causes a substitution effect between the amounts of inputs. Indeed:
\[
\left\{ \begin{array}{l}
\frac{ds^*_i}{d\tau} = s^*_i a_2 \left( \frac{1}{\sigma} \right) \left( \frac{d\sigma}{d\tau} \right) \\
\frac{ds^*_j}{d\tau} = -s^*_j a_1 \left( \frac{1}{\sigma} \right) \left( \frac{d\sigma}{d\tau} \right)
\end{array} \right. 
\]  \tag{B.2}

where \(\sigma\) is the ratio between the amount of inputs:

\[
\sigma \equiv \frac{s^*_i}{s^*_j} = \frac{a_1(w_2 + T_2 d_2)}{a_2(w_1 + T_1 d_1)} > 0
\]

Standard calculations reveal that \(\frac{d\sigma}{d\tau} = \sigma \chi\) with \(\chi \equiv \frac{\alpha_2 d_2 (w_1 + T_1 d_1) - \alpha_1 d_1 (w_2 + T_2 d_2)}{(w_1 + T_1 d_1) (w_2 + T_2 d_2)}\).

We can already notice that variation signs of optimal input quantities \((s^*_i, s^*_j)\) are opposite, regardless of the variation in carbon tax (see equation (B.2)). In other words, an increase in quantity \(s^*_i\) necessarily leads to a decrease in the other one \(s^*_j\) with \(i \neq j\). The sign of variation of each quantity \(s^*_i\) will depend on the sign of the parameter \(\chi\). Hence,

- If \(\chi > 0\), then the quantity of \(s^*_i\) increases and \(s^*_j\) decreases
- If \(\chi < 0\), then the quantity of \(s^*_j\) increases and \(s^*_i\) decreases

Suppose that supplier \(S_1\) is the most polluting per unit of shipped input, that is \((\alpha_2 d_2 < \alpha_1 d_1)\). Whereas \(\chi > 0\) (the quantity of \(s^*_i\) rises) if and only if

\[
d_1 < \hat{d}_1 \tag{B.3}
\]

with,

\[
\hat{d}_1 \equiv \frac{\alpha_2 w_1 d_2}{\alpha_1 w_2 + d_2 (\alpha_1 t_2 - \alpha_2 t_1)} > 0
\]

Considering the level of emissions per unit of the transported input, the expression (B.3) can be written as \(\alpha_1 d_1 < \alpha_1 \hat{d}_1\). As we have assumed \(\alpha_2 d_2 < \alpha_1 d_1\), then, \(\alpha_2 d_2 < \alpha_1 d_1 < \alpha_1 \hat{d}_1\). Finally, the following condition is obtained:

\[
\frac{\alpha_2}{\alpha_1} d_2 < d_1 < \hat{d}_1
\]
The same analysis can be performed under the assumption that supplier \( S_2 \) is the most polluting per unit of shipped input (\( \alpha_1 d_1 < \alpha_2 d_2 \)). We find the following condition:

\[
\hat{d}_1 < d_1 < \frac{\alpha_2}{\alpha_1} d_2
\]

**Appendix B.6. Transport modes analysis**

Let us examine the circumstances of **Lemma 3** with regard to transportation modes. Each mode is achieved by its transport unit cost \( t_i \) and emissions unit level \( \alpha_i \) per unit of input. Consider that the firm uses the same transport technology for its inputs supply, meaning that \( \alpha_1 = \alpha_2 \) and \( t_1 = t_2 \). Then, the most polluting input is necessarily the furthest input. In this case, \( \hat{d}_1 = \frac{w_1}{w_2} d_2 \); then, \( d_2 < \hat{d}_1 \) implies \( w_2 < w_1 \), and \( d_2 > \hat{d}_1 \) implies \( w_2 > w_1 \). Hence, from **Lemma 4**, the firm can increase the quantity for the furthest input when the carbon tax increases, provided that its relative price is high enough. This case is more likely to occur when the price gap (\( \frac{w_i}{w_j} \) with \( i \neq j \) such as input \( i \) is the most polluting and input \( j \) is the less polluting input) increases. The growth rate of the delivered price \( (w_j + T_j d_j) \) is higher for the cheaper input when the carbon tax increases. In other words, the relative delivered price of the more expensive input declines with carbon tax under these circumstances. This means that a higher tax burden leads to a convergence between the delivered input prices.

It follows that in the mono-modal transport scheme, when the carbon tax increases, the relative demand for the furthest and more polluting input \( i \) may increase if \( w_i - w_j > 0 \) with \( i \neq j \).

When focusing on a firm’s location, conditions of **Lemma 4** lead to: (a) If \( d_2 < d_1 < \hat{d}_1 \), then: \( y \in [0, d_1] \) such as: \( d_1 < \frac{\alpha_2}{\alpha_1} d_2 \) and (b) If \( \hat{d}_1 < d_1 < d_2 \), then: \( y > d_1 \) such as: \( d_1 > \frac{\alpha_2}{\alpha_1} d_2 \). This result and **Lemma 3** imply that

**Corollary 1.** For a mono-modal transport mode, when the carbon tax increases, the quantity of the more polluting input increases when the firm is located near the cheapest input source.
Whether the transport technologies for carrying inputs are different, the analysis is richest. Let us consider the configuration $\frac{\alpha_2}{\alpha_1} d_2 < d_1 < \hat{d}_1$ when $S_1$ is the more polluting supplier. Under this configuration, we must consider two cases.

**Case a:** When $\alpha_1 t_2 - \alpha_2 t_1 > 0$, $\frac{\alpha_2}{\alpha_1} d_2 < \hat{d}_1$ holds if and only if $w_2 < w_1$ and $d_2 < d_1$, with

$$d_2 \equiv \frac{\alpha_1 (w_1 - w_2)}{\alpha_1 t_2 - \alpha_2 t_1}.$$ 

Hence, as in the mono-modal transportation scheme, the quantity of the more polluting input increases when the carbon tax increases, provided that its price is high enough (high $w_1$), and the less polluting supplier ($S_2$) is not too far ($d_2 < d_2$).

**Case b:** When $\alpha_1 t_2 - \alpha_2 t_1 < 0$, $\frac{\alpha_2}{\alpha_1} d_2 < \hat{d}_1$ holds if and only if $w_1 < w_2$ and $d_2 < d_2 < \bar{d}_2$, with

$$\bar{d}_2 \equiv \left| \frac{\alpha_1 w_2}{\alpha_2 t_1 - \alpha_1 t_2} \right|.$$ 

This case reveals that the firm may increase the quantity of the more polluting input provided that its price is cheaper (low $w_1$), and that the less polluting supplier ($S_2$) is neither too far nor too close ($d_2 < d_2 < \bar{d}_2$).

Figure B.13: **Asymmetric effect of carbon tax.**

Figure B.13 illustrates the circumstances of the previous two cases when $S_1$ is the more polluting supplier per unit of shipped input. In the space $(d_1; d_2)$ is depicted the function...
$\hat{d}_1$ with case a (resp. case b) when $\alpha_1 t_2 - \alpha_2 t_1 > 0$ (resp. $\alpha_1 t_2 - \alpha_2 t_1 < 0$). The area cross-hatched with red fulfills the conditions of case a ($\frac{\alpha_2}{\alpha_1} d_2 < d_1 < \hat{d}_1$ and $d_2 < \tilde{d}_2$) and those for case b ($\frac{\alpha_2}{\alpha_1} d_2 < d_1 < \hat{d}_1$ and $d_2 < \tilde{d}_2$). Therefore, a firm’s location lying under these conditions of distances should increase the amount of the more polluting input (input 1) when the carbon tax increases.

A similar analysis is obtained when $S_2$ is the more polluting supplier. In Figure B.13, the area cross-hatched in green fulfills the conditions for which the firm increases the amount of the more polluting input (input 2) when the carbon tax increases.

This outcome demonstrates the asymmetric effect of a higher carbon tax by increasing the share of the most polluting input under some conditions when we allow substitution among inputs.

References


