Testing the CAPM under Asymmetric Information

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ABSTRACT

If investors are differently informed about the payoff of market-traded securities, then the traditional market portfolio is not a relevant benchmark for testing the CAPM. Each investor appraises expected returns and builds his optimal portfolio conditionally on his information. *Which proxy to use for conditional expected returns, and what is the relevant benchmark to consider for the conditional CAPM(s)*? Many CAPM empirical tests consider future realized returns as proxies for expected returns. Because realized returns embed an informational component, they are best proxies for expected returns conditioned on *all* available information, which means that we implicitly adopt the perspective of a perfectly informed (PI) investor. The contributions of this paper are to construct the PI's optimal portfolio and to consider this portfolio as the correct benchmark for testing the CAPM from the perspective of this investor. Our empirical results provide a more optimistic picture of the CAPM than previous studies.

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While the traditional Sharpe (1964) – (Lintner, 1965a,b) – Mossin (1966) capital asset pricing model (CAPM) assumes that investors are homogeneously informed about the future cash-flows generated by market-traded securities, noisy rational expectations equilibrium (REE) models consider the realistic assumption that investors have different pieces of information with respect to these cash-flows, and that equilibrium prices reveal partially this information.

Noisy REE models have been considerably developed over the past 40 years; yet, little research has been done with respect to their empirical and practical relevance. The main difficulty to make such models useful and applicable is that an econometrician, considered here as an "uninformed" investor, cannot observe the private signals received by other investors. Dybvig and Ross (1985) affirm: "*an uninformed observer using the tools of mean variance and security market line analysis to measure the performance of a portfolio manager who has superior information is unlikely to be able to make any reliable inferences*".

The traditional market portfolio is the optimal portfolio for every investor in a context of perfect markets and homogeneous information. On the contrary, if investors have different pieces of information, they choose their optimal portfolio by conditioning the securities' expected returns and variance on public and private information; the traditional market portfolio is no more a universal benchmark. Each investor has his own efficient frontier, asset-pricing model, and optimal portfolio conditional on his information set. Biais, Bossaerts, and Spatt (2010) highlight a consequence of heterogeneous expectations on portfolio choice: uninformed investors suffer a winner's curse problem, investing more (resp. less) in assets in which they are more (resp. less) optimistic than informed agents. Based on a noisy REE model, Biais et al. (2010) implement a strategy for uninformed investors in order to form their optimal portfolio using the information partially revealed by prices on the US market. They find that prices do contain useful information and the uninformed investors' portfolio generates higher returns than a buy-and-hold strategy for the same level of risk.

The empirical evidence on the traditional CAPM is not very supportive. Several researchers (e.g., Fama and French, 1992) have acknowledged a flat relation between beta and average stock returns; others even find that betas are lower for stocks generating higher future returns. Harvey, Liu, and Zhu (2016) have counted more than three hundred anomaly variables which help forecasting returns. It follows a general consensus: the traditional CAPM is unable to satisfactorily explain the cross-section of average stocks returns. This comes as an opposition between the financial theory and the empirical observations.

In 1977, Roll formulated his well-known critic that "*the theory is not testable unless the exact composition of the true market portfolio is known and used in the tests*". A related critic can be formulated in a context of asymmetric information: to test the CAPM, the econometrician should first know (1) what is the conditional CAPM that can be tested with the available data, and (2) what is the relevant portfolio to use as a benchmark. Regarding the first point, most CAPM tests use realized stock returns as a proxy for expected returns. We claim that, under asymmetric information, realized stock returns are a relevant proxy for expected returns as perceived by perfectly informed (PI) investors. Indeed, realized returns are the sum between two components: an informational component, which is perfectly known by these investors, and a random component, which is not known, also called sometimes "residual uncertainty". It follows that an empiricist using future realized returns as proxies for expected returns *implicitly* considers the CAPM from the perspective of a PI investor.

Regarding the second point, the problem is that the traditional market portfolio, widely used in CAPM tests, is not the relevant portfolio from the perspective of a PI investor; relative to the market portfolio, this investor over-weighs securities on which he has positive information, and will under-weigh those on which the information is unfavorable. Our contribution is to exploit noisy REE models (see for example Grossman and Stiglitz, 1980; Admati, 1985; Biais et al., 2010) in order to construct this portfolio. Our methodology builds on Biais et al. (2010) who project stock returns onto prices of relevant portfolios in order to build the optimal portfolio held by an uninformed investor. The main difference in our paper is that we adopt the perspective of a PI investor, not that of uniformed investors. We extract both public and private signals by projecting stock returns on beginning-of-period stock prices and stock trades. The volume traded, jointly with beginning-of-period prices, represent, according to noisy REE models, a sufficient statistic allowing the econometrician obtaining all the available information. Indeed, in these models the informational component in stock returns writes as a linear function of these two variables.

Most REE models consider supply uncertainty as noise preventing prices from fully revealing all the available information. These models predict that an investor that is able to observe prices and volumes traded would be able to extract all the relevant information. We therefore use the volume traded on the market concomitantly with stock prices in order to obtain the private signals received by a PI investor and to implement a price- and volume-contingent strategy for building his optimal portfolio. This portfolio is then used to compute the "full-information" betas of common stocks, that is, the betas from the perspective of a PI investor, and then to test the CAPM from the perspective of this investor. Our empirical results regarding the conditional CAPM provide a more optimistic picture than those obtained for the traditional CAPM.

For Jagannathan and Wang (1996), the poor ability of the traditional CAPM to explain the cross-sectional variations in average stock returns comes from the fact that beta varies over time. However, testing a dynamic version of the CAPM (i.e., with betas and expected returns allowed to vary over time) is challenging. The authors derive, from the conditional CAPM, an unconditional two-factor model, with the default premium as a conditioning factor. Their results show that the dynamic CAPM provides a better description of average stock returns than the static CAPM. For Lewellen and Nagel (2006), the failure of the unconditional CAPM cannot only be explained by time variations in beta. The observed pricing errors with the unconditional CAPM and asset-pricing anomalies such as the value effect are simply too large for that.

Our main contribution is to exploit REE models in order to derive the correct tools of mean variance and security market line analysis needed in order to provide a correct test of the CAPM. More precisely, our contributions reside in (1) implementing an empirical strategy for constructing the relevant benchmark portfolio from the perspective of such an investor and (2) providing an empirical test in order to verify the relevancy of the "full-information" CAPM based on our new benchmark.

The paper is organized as follows. The next section presents our theoretical background for building our new benchmark portfolio and testing the CAPM based on this benchmark. Section II presents our econometric model and our empirical results are discussed in Section III. Section IV concludes.

I. Theoretical Background

A. *The Informed Investor's Portfolio*

The theoretical justification of our strategy for building the PI investor's portfolio may be illustrated using the framework of noisy REE models considering asymmetric information between investors. In general, such models are classified in two main categories. The first one considers that the information is *stricto sensu* asymmetric, that is, all the available information on the revenues generated by market-traded securities is detained by just one category of investors, designated as "informed investors". The seminal papers of Grossman and Stiglitz (1980) belongs to this category, but considers markets with just one risky security; their model has been generalized in multi-asset frameworks (e.g., Kodres and Pritsker, 2002; Jimenez-Garcès, 2004). The second category of models consider many categories of investors that detain different pieces of information on the revenues generated by market-traded securities. In this category, seminal papers conceived in a one-risky-asset framework such as Hellwig (1980) or Verrecchia (1982) have been generalized in a multi-risky-asset framework by Admati (1985), Biais et al. (2010) among others.

One important issue in noisy REE models is to find a closed-form solution for the equilibrium price of market-traded securities. Under the presence of noise, equilibrium prices reveal partially, not perfectly, the available information. In most models, the noise is induced by random per capita supply of risky assets per investor, in many cases denoted by $\tilde{Z}$, which is modeled as a random variable having a normal distribution. This source of randomness plays two key roles. It prevents prices from fully revealing all the available information, but it also prevents agents from refusing to trade with better informed traders (see Milgrom and Stokey, 1982). In practice, considering random supply is realistic since, although the number of shares outstanding is a fixed amount distributed among the market participants, not all shares are traded, but only an amount that is unknown ex ante.

In general, these models assume that traders have negative exponential utility functions, which imply constant absolute risk aversions. This hypothesis simplifies the analyses since an asset's demand depends upon a trader's beliefs, not directly on his wealth. The private information received by investors is modeled as a private signal that they receive about the securities' payoffs, that we denote by $\tilde{F}$, considered as a random variable having a normal distribution and joins the other distributions considered by the models.

One specificity of noisy REE models is that they are very complex and do not allow finding a closed-form solution[1] for equilibrium prices unless restrictive conditions apply. Under certain conditions studied specifically by each paper, the equilibrium price vector of risky assets $\tilde{P}^0$ may write, in many such models, as a linear function of a random variable that we designate as "information" and denote $\tilde{F}_I$, and the random supply $\tilde{Z}$:

$$\tilde{P}^0 = A_0 + A_1 \cdot \tilde{F}_I - A_2 \cdot \tilde{Z} \tag{1}$$

where A_0 is a $(N \times 1)$ vector, and A_1, A_2 are two $(N \times N)$ matrices, with A_1 and A_2 being nonsingular matrices. For example, in Admati (1985), the information component is represented by the future cash-flow $\tilde{F}$ generated by the securities on the market (in this model, a PI investor would know exactly this cash-flow since the model considers that there is no residual uncertainty on the market). In Grossman and Stiglitz (1980), the informational component $\tilde{F}_I$, denoted by $\tilde{\theta}$ in their paper, represents a signal on the future cash-flow generated by the security, $\tilde{F}_I = \tilde{\theta} + \tilde{\epsilon}$, where $\tilde{\epsilon}$ represents a residual term.

Consider the perspective of a PI investor in these settings. This investor is defined as one receiving all the signals received by all investors that trade on the market. In Grossman and Stiglitz (1980), this investor is considered explicitly, since in this model there only two categories of investors: informed and uninformed ones. In Admati (1985), who considers dispersed information, each investor a observes a private signal $\tilde{Y}_a = \tilde{F} + \tilde{\varepsilon}_a$, where $\tilde{\varepsilon}_a$ is a disturbance term (a lower variance of this term means that the signal is more precise). In this model, since there are an infinity of investors and since private signals are independent, the residual terms sum up to zero, which implies that the equilibrium prices writes as in the equation specified above. In Admati (1985), the PI investors would be an hypothetical investor that would receive all the private signals $\tilde{Y}_a$ received by all the various investors on the market. Since there is no residual uncertainty, this investor would know exactly the future cash-flow generated by every security on the market, which means that he would bear no risk.

In practice, it is impossible to obtain all the relevant information about the future cash-flow generated by market-traded securities. According to noisy REE models, a part of the relevant information is reflected by prices; under the presence of noise, prices cannot reveal perfectly all the available information. In this case, an uninformed investor would extract the information from prices and would obtain a better performance than an uniformed investor that does not condition

[1]To solve for the equilibrium, the method of undetermined coefficients is the standard approach, working as a "guess and verify" solution method. First, we guess a functional form for the price function: the hypothesis is that the price is a linear function, increasing in $\tilde{F}$ and decreasing in $\tilde{Z}$. Then we use it to figure out what this price implies for agents' risky assets demands, deriving the posterior beliefs of investors. Given that agents learn something about $\tilde{F}$ from observing asset prices $\tilde{P}^0$ and that at the same time assets prices are influenced by their demand, this is a fixed point problem. $\tilde{P}^0$ and $\tilde{D}_a$ need to be solved jointly. Substituting the demand functions into the market clearing condition gives the actual price function after some mathematical manipulations. Undetermined coefficients are solved in a system of equations, one for each coefficient sets equal to zero. The REE is the fixed point of the mapping of the conjectured price relationship onto the actual price functions, i.e. the point at which the two equations coincide.

his portfolio on this type of information. This problem has already been analyzed by Biais et al. (2010) who show that a price-contingent strategy performs better than a buy-and-hold strategy.

Our strategy for building the PI investor's portfolio exploits, for the first time as far as we know, a prediction of noisy REE models that is simple from a theoretical perspective, but which is less simple to implement empirically. Our reasoning is as follows. An investor that is able to observe both $\tilde{P}^0$ and $\tilde{Z}$ would be able to infer all the relevant information on $\tilde{F}_I$ because $\tilde{F}_I$ is a linear function of $\tilde{P}^0$ and $\tilde{Z}$ and, in existing models, the matrix A_1 is nonsingular: $\tilde{F}_I = A_1^{-1}(\tilde{P}^0 - A_0 + A_2 \cdot \tilde{Z})$. Hence, an investor observing the equilibrium price $\tilde{P}^0$ and the supply $\tilde{Z}$ would have a sufficient statistic for the aggregate information set. In other words, the price and the supply summarize *all* the information throughout the economy.

The PI investor's demand is therefore a function[2] $D(\tilde{P}^0, \tilde{F}) \equiv D(\tilde{P}^0, \tilde{Z})$. This is a function of the equilibrium price and his signal, and equivalently a function of the equilibrium price and the supply. The econometrician can thus build the informed investor's portfolio even if he does not observe private signals; extracting information from prices and supply is sufficient to acquire all the relevant information and build this portfolio. Such a strategy may be implemented by projecting securities' returns on the prices and a proxy of supply using an econometrical methodology similar to the one put forward by Biais et al. (2010). The problem is that this strategy is not investable ex ante because the supply is assumed to be known with delay.

From the point of view of the econometrician, the supply is unknown ex ante. The important question to answer is about the relevant proxy to use for supply. In our study, we use the traded volume over the month preceding the portfolio rebalancing decision. This data is available ex ante but not to everyone, since volumes become publicly available with delay. The fact that the supply is unknown for uninformed investors allows maintaining a partially revealing equilibrium.

B. *The Conditional CAPM*

While in the traditional CAPM holding the market portfolio is optimal, in a situation of informational asymmetry the optimal portfolio of investors is a function of information. A change in the expectation of the signal received by investors leads them to modify their portfolio, even if the supply does not change. A buy-and-hold strategy is then no longer optimal. Considering the same context, Biais et al. (2010) consider a fictitious representative agent whose beliefs are a weighted average of the informed and uninformed agents' beliefs and derive the CAPM from the point of view of this representative agent.

The conditional CAPM model from a PI investor's perspective in a context of information asymmetry can be relatively easily derived in the context of existing noisy REE models. The classical assumption is that investors have negative exponential utility functions, which imply constant

[2] At first sight, as his signal is perfectly precise, this perfectly informed investor should not need to rely on the equilibrium price to update his beliefs. Being only partially revealing, it is necessarily less informative. However, in this Walrasian equilibrium concept, the tâtonnement process is central: demands and prices adjust iteratively with no trade until the price that will clear the market is found. As the total demand matches supply only after iterations, he does observe P^0.

absolute risk aversions. By denoting investors absolute risk tolerance coefficient by ρ, the demand function of the PI investor for an individual asset i writes:

$$D_I = \rho Var_I \left[\tilde{F}_i - R\tilde{P}_i^0\right]^{-1} E_I \left[\tilde{F}_i - R\tilde{P}_i^0\right] \tag{2}$$

where: the subscript I indicates that the variance and the expectation are conditioned on the information possessed by the PI investor; $R = 1 + r_f$, where r_f is the risk-free rate of return; $\tilde{F}_i$ is the future cash-flow generated by asset i; and $\tilde{P}_i^0$ is the equilibrium price of asset i. It follows that the conditional expected return, expressed as differences in prices, writes as follows,

$$E_I \left[\tilde{F}_i - R\tilde{P}_i^0\right] - r_f E \left[\tilde{P}_i^0\right] = \rho^{-1} Cov_I \left[\tilde{F}_i - R\tilde{P}_i^0, \tilde{F}_{opt.I} - R\tilde{P}_{opt.I}^0\right] \tag{3}$$

where the subscript *opt.I* designates the optimal portfolio of the PI investor. After some classical mathematical manipulations (see Appendix A), we obtain the following specification of the conditional CAPM:

$$E_I(\tilde{r}_i) - r_f = \beta_{I,i}(E_I(\tilde{r}_{opt.I}) - r_f) \tag{4}$$

where $\beta_{I,i} = \dfrac{Cov_I(\tilde{r}_i, \tilde{r}_{opt.I})}{Var_I(\tilde{r}_{opt.I})}$

An important aspect is that the beta conditional on PI investors' information at time t for a stock i, $\beta_{I,i}^t$, designated in this article as the "full-information" beta, is obtained, as in the traditional CAPM, by regressing stock i's return onto the optimal PI investor's portfolio. According to this CAPM specification, a higher such beta is expected to be associated with higher expected returns, the latter being conditioned on the information of the PI investor.

In a symmetric information context, the beta, obtained by using a value-weighted stock index as proxy for the market portfolio, captures the systematic risk. By contrast, in presence of informational asymmetries, the beta calculated in the traditional way is relevant only if we adopt the perspective of a hypothetical investor holding optimally the market portfolio. The problem is that the CAPM cannot be correctly tested from the perspective of such an investor because it seems a priori difficult to find a relevant proxy for his conditional expected return and variance.

II. Econometric Approach

A. Data

We focus on the French market. French common stocks are listed on Euronext.liffe Paris. We consider monthly stock returns on the period from August 1991 to December 2014. Relatively to daily or weekly returns, the monthly frequency diminishes the number of observations but is less concerned by microstructure problems, such as non-synchronous or "thin" trading, than weekly or daily returns. More frequent adjustments would also increase transaction costs.

Adjusted and quoted prices, the number of existing common stocks, the number of common stocks traded and other relevant data are extracted from Datastream and Thomson ONE. We exclude cross-listed stocks and those which are not primary quote, a choice that does not influence qualitatively our results. Daily traded volume is obtained by multiplying the number of shares traded a given day by the unadjusted closing price at the end of this day. Monthly traded volume (hereafter simply "volume") is the sum of daily volume during the month t first and last days. The risk-free rate is the French one-month Treasury bill.

As already acknowledged by Biais et al. (2010), it would be impossible to form the PI investor's portfolio by analyzing individual stocks. For this reason, we group stocks into portfolios as in Biais et al. (2010) and then project their returns on beginning-of-period prices and volumes traded in order to implement our price- and trade- optimal strategy as per the PI investor's perspective. Precisely, we use a Fama-French (FF) 2×3 sort on size and book-to-market factors to form six portfolios in order to allow comparisons between our results and the existing literature.

In accordance with Fama-French procedure, Market Equity (ME) is defined as the unadjusted share price multiplied by the number of existing ordinary shares. Book common equity (BE) is defined as the book value of stockholders' equity, plus balance-sheet deferred taxes and investment tax credit (if available), minus the book value of preferred stock. We use ME and BE at the end of December of year $t-1$ to compute the book-to-market ratio (BtM), and ME at the end of June of year t to measure firm size. To be included a year, a firm must have ME data available at these two dates, and a positive BE as well. To avoid survivorship bias, we do not include firms unless they have two years of available BE. Six FF portfolios are formed at the end of June of year t, to be sure that the BE will be known, even for firms which fiscal year ends at the end of March of year t. They are obtained by crossing the size and BtM criteria (2×3). We compute the median size to split the sample into two groups (small and big) at the end of June of year t. The ranking on BtM is realized at the end of December of year t by computing two quantiles (30 and 70%) to break the sample into three groups (low, medium and high). The 6 FF portfolios are then monthly value-weighted from July of year t to the end of June of year $t+1$. Please refer to Fama and French (1993) for more details.

To check the robustness of our results, we also group stocks based on industries, as in Burlacu, Fontaine, Jimenez-Garcès, and Seasholes (2012). In this case, we assign each stock to an industry portfolio based on its four-digit standard industrial classification (SIC) code according to the Fama-French five industries classification (more details are provided in Appendix B). SIC codes are collected with Thomson ONE, and primarily completed with Osiris, or manually when they were not available in these databases. We chose a five-industry classification in order to limit the number of explanatory variables in our regression to 10 (5 portfolios for prices and volumes).

The final sample contains 1,615 firms which meet all the requirements, spanning 471 four-digit SIC industries, over 281 months. To test whether the CAPM holds for this sample, we retain only firms with a complete data history over the 4 years of the portfolio formation period and the 5 years of the estimation period. Over the 1,615 firms, only 251 meet this requirement.

B. Methodology for Building the Informed Investor's Portfolio

Our methodology for constructing PI investor's optimal portfolio considers projecting returns on beginning-of-period prices and volumes of relevant portfolios and then applying the mean-variance methodology used in Biais et al. (2010).

The French market index used in our study is based on the sample of 1,615 firms which meet all the requirements explained in the previous section. The return of this index is calculated by value-weighting the sample securities. Figure 1 and Figure 2 compare our index to the SBF 250.

[Place Figure 1 about here]

[Place Figure 2 about here]

The six FF portfolios are value-weighted. Standard tests highlight the presence of a unit root in prices, which leads us to control for stationarity by calculating relative prices as in Biais et al. (2010). The relative price of portfolio i at the end of month t is:

$$P_i^t = \frac{P_i^{t-1} \times (1 + r_i^t)}{\sum_{j=1}^{6} P_j^{t-1} \times (1 + r_j^t)}$$

The volume associated to a given portfolio is calculated in two steps. First, we cumulate monthly volumes, expressed in euros, of the common stocks belonging to each portfolio i in order to obtain the "absolute" euro-volume traded during the period. Second, we calculate the relative volume Z_i^t of the portfolio i during month t by dividing the absolute volume obtained in the first step by the volumes of the six FF portfolios. Figure 3 presents the distribution of the monthly returns of the six FF portfolios from January 1991 to December 2014.

[Place Figure 3 about here]

We project portfolios returns during period t onto the beginning-of-period portfolios' prices and volumes[3]. Coefficients are estimated with the ordinary least squares (OLS) method, since econometric tests did not detect autocorrelation nor heteroscedasticity problems in individual error terms. For each one of the 6 Fama-French portfolios, we perform the following regression:

$$r_{FFj}^t = \sum_{j=1}^{6} \beta_j \cdot P_{FFj}^{t-1} + \sum_{j=1}^{5} \beta_{j+6} \cdot Z_{FFj}^t + \varepsilon_i^t$$

A full rank issue induced by the use of relative prices and volumes leads us to omit a volume (please consult Appendix C for more information).

[3] Indeed, in a multi-asset framework, payoffs and supplies are correlated across assets. This interaction is captured by the projection of the return of a security block onto its own price and own volume but also with the prices and volumes of other security blocks.

We use 60 observations for our regressions, so the first date at which we form our PI investor's portfolio is August 1, 1996. Then we rebalance our portfolio each month. The variance-covariance matrix is computed from the six series of residuals. The weight q_i^t of the FF portfolio i within the informed portfolio at the beginning of month t is obtained by maximizing[4] the expected return for the same level of risk as the market index, i.e.:

$$\max_{q^t} E(q^{t\prime} r^t | P^{t-1}, Z^t) \text{ subject to } \sigma(q^{t\prime} r^t | P^{t-1}, Z^t) = V_{index}^{t-1}$$

The ex ante market index volatility V_{index}^{t-1} is estimated as the root mean squared difference between its return and that predicted by the following regression through the 60 months prior to the target month:

$$r_{index}^t = \beta_1 \cdot P_{FF1}^{t-1} + \beta_2 \cdot P_{FF2}^{t-1} + \beta_3 \cdot P_{FF3}^{t-1} + \beta_4 \cdot P_{FF4}^{t-1} + \beta_5 \cdot P_{FF5}^{t-1} + \beta_6 \cdot P_{FF6}^{t-1} + \varepsilon_i^t$$

C. Methodology for Testing the Conditional CAPM

In order to test the full-information CAPM, we use the two-stage cross-sectional regression approach of Fama and MacBeth (1973), hereafter "FM". First, market betas are estimated for each common stock by regressing its return on the return of the PI investor's portfolio, then we examine their explanatory power on future realized stock returns, which are considered as a proxy of the expected returns conditional on the PI investor's information.

Using estimates instead of real market risk premiums inevitably induce an error-in-variables (EIV) bias. FM procedure implies working with a large number of individual assets grouped in portfolios, rather than individual assets, to reduce this EIV problem. Securities are assigned to portfolios according to their beta, in order to minimize within-portfolio variation in betas. As in Fama and French (1992), we refer to these betas as "pre-ranking" betas, and estimate them in a preliminary step.

More specifically, following Fama and MacBeth (1973), we form 20 portfolios from pre-ranking individual betas, obtained by regressing 4 years of monthly excess (over the risk-free rate) returns of each stock (with dividends reinvested) on the excess return of the market portfolio. We limit the portfolio formation period to 4 years, compared to 7 years in FM methodology, in order to keep a sufficiently large sample. Then, post-ranking individual betas, $\hat{\beta}_i$, are calculated by using the next 5 years of monthly asset excess returns, and updated yearly by extending the time-series length with one to three additional years. Portfolios are equally-weighted. Portfolio betas, $\hat{\beta}_{p,t}$, and portfolio returns, $R_{p,t}$, are monthly adjusted for delisted stocks or stocks with missing returns. Finally, for each of the following 48 months, the following cross-sectional regression is performed:

$$R_{p,t} = \hat{\gamma}_{0,t} + \hat{\gamma}_{1,t} \cdot \hat{\beta}_{p,t-1} + \hat{\gamma}_{2,t} \cdot \hat{\beta}_{p,t-1}^2 + \hat{\gamma}_{3,t} \cdot \bar{s}_{p,t-1}(\hat{e}_i) + \hat{\eta}_{p,t} \quad (5)$$

[4]We use Markowitz's solution to this constrained maximization problem to find efficient portfolios. A Lagrangian procedure delivers the weights which maximize portfolio expected return for a given level of risk.

The values of these coefficients will provide fundamental indications about the validity of the CAPM, as demonstrated by Fama and MacBeth (1973). They derive three testable conditions common to Sharpe–Lintner (S–L) and Black versions[5], plus one condition specific to the S–L version:

- S–L hypothesis: with risk-free borrowing and lending, the expected returns on zero-beta assets[6] equal the risk-free rate[7], and thus the intercept should equal the risk-free rate.
- C1: higher risk should be associated with higher expected return.
- C2: the relationship between the expected return of a security and the covariance with the market portfolio is linear.
- C3: there is no other measure of risk than the market beta, β.

The intercept $\hat{\gamma}_{0,t}$ represents the pricing error, i.e. the cross-section of average stock returns left unexplained by the model. If $E(\tilde{\gamma}_{0,t}) = 0$ then the S–L hypothesis is supported by the data[8].

The coefficient that is central in our study, $\hat{\gamma}_{1,t}$, reflects the ability of the market beta as measured by the PI investor to explain the cross-section of average returns. We expect its value to be significantly positive using the optimal informed investors' portfolio (C1). Indeed, in an asymmetric information context, whether the conditional CAPM works in theory just as well for informed investors as for uninformed investors or from the point of view of any other investor, the fact that realized returns are used in practice as proxy for expected returns makes the conditional CAPM only usable by PI investors. To compare results, we will nonetheless test three possibilities for estimating the beta: with the informed investor's portfolio, the uninformed investor's portfolio, and the traditional market portfolio.

The coefficient $\hat{\gamma}_{2,t}$ tests whether the relationship between expected returns and β is linear (C2). If $E(\tilde{\gamma}_{2,t}) = 0$, then the linearity hypothesis is not rejected.

Finally, $\hat{\gamma}_{3,t}$ indicates whether there are other measures of risk, in addition to β, that contribute systematically to observed average returns (C3). We expect its value to be non-statistically different from zero while using a traditional index as proxy for the market portfolio or the optimal uninformed investors' portfolio, and significantly different from zero while using the optimal informed investors' portfolio. Indeed, we argue that using a market index not only estimates the systematic risk but also the information risk, while using the optimal informed investors' portfolio, not subject to the information risk, allows estimating solely the systematic risk. In the first case, the systematic risks

[5]The Sharpe–Lintner version of the CAPM allows unrestricted borrowing and lending at a risk-free rate. In comparison, the Black version (1972), more general, releases this assumption. Hence, in the absence of a riskless asset, the only requirement is that the expected market return be greater than the expected return on zero-beta assets.

[6]Fama and French (2004) give the following definition: "*A risky asset's return is uncorrelated with the market return—its beta is zero—when the average of the asset's covariances with the returns on other assets just offsets the variance of the asset's return. Such a risky asset is riskless in the market portfolio in the sense that it contributes nothing to the variance of the market return*".

[7]This leads to the familiar Sharpe–Lintner CAPM equation, equal to the minimum variance condition of the market portfolio but with the risk-free rate instead of the expected return on zero-beta assets.

[8]In the Sharpe–Lintner version of the CAPM, the intercept should equal the risk-free rate, denoted r_f, i.e., using stock returns, $E(\tilde{\gamma}_{0,t}) = r_f$. Equivalently, we test $E(\tilde{\gamma}_{0,t}) = 0$ using excess stock returns.

and information risks are thus incorporated into the model, while in the second situation only the systematic risk is present, leaving the information risk outside the model.

To further correct for EIV bias, the coefficients from the cross-sectional regressions are averaged over time using the Litzenberger and Ramaswamy (1979) method (hereafter "LR" correction). Coefficients are weighted by the inverse of their standard error when summing across the cross-sectional regressions, in order to place more (resp. less) weight on parameters that are estimated more (resp. less) precisely.

III. Empirical Results

A. *The Informed Investor's portfolio*

Table II provides descriptive statistics of the FF portfolios used in the process of building the PI investor's portfolio. We notice that on the French market small stocks provide less returns on average and are less volatile than big stocks. The portfolio S/H is the exception since its return is higher than B/H, for a lower risk. Hence, contrary to expectations, the Small Minus Big (SMB) premium documented by Fama and French (1993) is negative. Fama and French (2012) also find a negative small firm effect in Europe from November 1990 through March 2011. As for the value premium on the French market, we find a positive High Minus Low (HML) average return.

[Place Table II about here]

Figure 4 displays the evolution of the (relative) prices of the six FF portfolios. The value of the small stocks is initially lower than their counterpart with big capitalizations since they are initialized by value weighting. Contrary to Biais et al. (2010), this trend does not reverse because of the big firm effect discussed above.

[Place Figure 4 about here]

At equal risk[9], the average monthly return of the informed portfolio over our sample period is 1.36% (17.58% annually), that of the uninformed portfolio is 1.15% (14.74% annually), and that of the market index is 0.83% (10.99% on an annual basis). As expected, the informed portfolio exhibits the highest performance.

Figure 5 presents the difference between the Sharpe ratio of the informed portfolio and the Sharpe ratios of the market portfolio and that of the uniformed investor's portfolio. The partial z-statistic, determined as in Biais et al. (2010), presented in Figure 6, allows determining the regions of acceptance of the null hypothesis with 95% confidence that the Sharpe ratios of the informed and uninformed portfolio are not statistically different. Figure 7 presents comparison results between the informed and uninformed investor's portfolio and indexing portfolios using value, momentum

[9]Ex post volatilities are matched over this period by determining the proportion to invest in the market portfolio and the corresponding strategy (please refer to Biais et al., 2010 for more details).

and size enhancements. In general, these figures show a better performance of the PI investor's portfolio over the others.

[Place Figure 5 about here]

[Place Figure 6 about here]

[Place Figure 7 about here]

Table III presents the performance of the uninformed and that of the PI investor's portfolio as appraised by the Treynor ratio and the alpha obtained from projecting the returns of these two portfolios onto the return of the market index, the returns of the Fama and French (1993) zero-investment portfolio SMB (Small Minus Big) and HML (High Minus Low), and the Carhart (1997) zero-investment portfolio UMD (Up Minus Down)[10].

[Place Table III about here]

The alpha of the two portfolios are positive and significant in most performance model specifications. The alpha of the PI investor's portfolio is higher, which conforms to our expectations.

Table IV gives the average sensitivities of the uninformed and informed portfolios to the six FF portfolios. We notice that informed investors invest more in small capitalization stocks, shorting B/M and B/H stocks. Stocks with both small capitalization and high value, which present as noted in the descriptive statistics the best risk/return profile, are more heavily in the uninformed and informed portfolios. In line with theory[11], informed investors hold more small-cap stocks, which might be more concerned with informational asymmetries.

[Place Table IV about here]

Forming the PI investors by projecting returns on the prices and volumes of the industry portfolios as defined previously do not change qualitatively the picture. Figure 8 presents the value of 1€ invested on July 31, 1996, with monthly returns reinvested, in the PI investor's portfolio, the uninformed portfolio, and the market portfolio. Again, the PI investor's portfolio dominates the other portfolios.

[Place Figure 8 about here]

[10]These results are provided for illustration purposes. Indeed, according to our initial argument, the market index has no relevance as a performance measure in a context of information asymmetry between investors.

[11]See for example Atiase (1985), Bhushan (1989) or Hasbrouck (1991).

B. FM Tests of the Conditional CAPM

According to our initial arguments, using realized returns as proxies for expected returns for CAPM tests assumes implicitly that we are adopting the perspective of a PI investor. The relevant beta of common stocks should be estimated by projecting their returns on the returns of the PI investor's optimal portfolio, not on the returns of the traditional market portfolio. In this case, the CAPM test should consider the link between such a "full-information" beta and future returns. This type of beta is purely a measure of systematic risk; being fully informed, the PI investor does not incur "information risk". According to the theory, the relationship between the full-information beta and future returns should be positive and significant.

The beta of individual securities is calculated with the Fama and French (1992) procedure by forming 20 portfolios as described previously. Please consider Table V for additional information on these portfolios (such as their duration, number of securities per portfolio, and the average beta for the three major portfolios considered in this study).

[Place Table V about here]

Table VI presents the results of our "full-information" CAPM test using the FM procedure for individual securities. Consistent with our expectations, the coefficient γ_1 associated to the market beta β is not significantly different from zero if we use the value-weighted index as a benchmark. The coefficient γ_3 is also insignificantly different from zero. This indicates that there isn't any other risk measure, in addition to β, that systematically contributes to average returns.

[Place Table VI about here]

If the optimal informed investors' portfolio is used as a benchmark, γ_1 is higher, with a t-stat[12] of 1.543, and γ_3 is statistically different from zero, which indicates that a risk premium is missing in the model. The R^2 is also lower (5.4% with the conditional portfolio compared to 9.5% with the value-weighted stock index)[13]. The coefficients γ_0 and γ_2 are not significantly different from zero in the two cases, implying that the S–L and linearity hypotheses are not rejected.

In addition, we perform the CAPM test using the FM procedure and the uninformed investors' portfolio as a benchmark. This analysis is provided for illustration purposes only. Indeed, as stated before, the uninformed investor's portfolio is not a relevant benchmark for CAPM tests using realized returns as expected returns.

[12] t-statistics are computed as:

$$t\left(\bar{\bar{\gamma}}_j\right) = \frac{\bar{\bar{\gamma}}_j}{s\left(\bar{\bar{\gamma}}_j\right)/\sqrt{n}}$$

[13] Cross-sectional R^2 is calculated as:

$$R^2 = 1 - \frac{Var_c\left(\bar{\bar{\eta}}_j\right)}{Var_c(\bar{R}_p)}$$

where Var_c denotes a cross-sectional variance, and variables with bars over them denote time-series averages.

Table VI, Panel B, shows that the C3 hypothesis cannot be rejected, so no other factor explaining systematically stock returns is missing. Consequently, this portfolio is subject to information risk. We also notice that the R^2 is low (0.1%).

Since we use a value-weighted index, we also test if our results are not sensitive to the weighting scheme. To this end, we perform the FM tests with an equally-weighted index, following Black, Jensen, and Scholes (1972) or Fama and MacBeth (1973). Table VII compares the use of these two indices as proxies for the market portfolio. Our conclusions remain qualitatively unchanged. The beta on the equally-weighted index has the wrong sign, but this beta is not significant.

[Place Table VII about here]

C. Robustness Tests

This section presents the results of the CAPM tests by considering portfolios based on firm characteristics that are known to be correlated with expected returns. First, we form 100 portfolios on the basis of size and market betas (10×10), well known to lead to a CAPM rejection (Fama and French, 1992). More specifically, stocks are sorted into size deciles, subdivided into beta deciles, estimated by using the value-weighted stock index over 24 to 60 months of past returns. Portfolios are formed each year, and stock returns are equally-weighted.

Time-series averages of portfolio returns are given in Table VIII, Panel A, where $Si/\beta j$ denotes the portfolio whose size is in the ith decile and market beta is in the jth decile.

[Place Table VIII about here]

Returns range from a low of 0.16 percent to a high of 2.35 percent per month. The dispersion is higher than reported in Fama and French (1992, Table I[14]) and Jagannathan and Wang (1996, Table I). We again observe a negative small firm premium. When we compute the average return for the five first size deciles across the betas, we find 0.88%, compared to 0.91% for the five last deciles, i.e. a size premium of −0.03%. This is equivalent to calculating $SMB = (S1/B1 + S1/B2 + \ldots + S5/B10)/50 - (S6/B1 + S6/B2 + \ldots + S10/B10)/50$. Note that the sample is not exactly the same than the one used to build the optimal informed investors' portfolio, the requirements for data availability being different. We have 653 stocks/year in average for the 6 FF portfolios formed on size and book-to-market, compared with 566 stocks/year in average for the 100 FF portfolios formed on size and beta.

The betas of the portfolios are presented in Panel B. They range from a low of 0.17 to a high of 1.81. We calculate the size of a portfolio as the equally-weighted average of the logarithm of market value of stocks (in million euros). The time-series averages of portfolio size are presented in Panel C. They range from a low of 1.46 to a high of 9.71.

[14] FF (1992) and JW (1996) only use returns from nonfinancial firms, while we also use returns from financial firms to be consistent with the optimal informed investors' portfolio, built from all firms.

Results of the FM tests are presented in Table IX.

[Place Table IX about here]

The average slopes allow determining which explanatory variables on average have non-zero expected premiums. The coefficient λ_1 is negative and not significant. By sorting stocks on size, the market does not help explain average stock returns, whether for the traditional (unconditional) CAPM, as in Fama and French (1992), or after conditioning the CAPM on information.

IV. Conclusion

As econometricians, we face the same problem as uninformed investors, that is, ex ante we are unable to perfectly extract information from prices. However, an ex post analysis is conceivable. Relying on REE models under asymmetric information, this paper starts by providing a procedure for building the optimal portfolio of a PI investor on the market. The intuition underlying our methodology is that, under the assumption that noise resides in random exogenous supply, observing aggregate supply controls noise and allows to perfectly infer all the relevant available information from prices. Therefore, since we do know ex post the supply realization (using volumes as proxy), we can be on a par with informed investors and build their portfolio at the aggregate level. Using data on the French market, our performance comparison tests reveal that the informed portfolio dominates the price-contingent strategy of the uninformed portfolio as proposed by Biais et al. (2010), both portfolios outperforming the market index.

The second contribution of our paper is to provide a CAPM test that is justified by REE models under asymmetric information. Such models predict that the expected return of a security conditional on an investor's information is higher for securities which beta, conditional on this information, is higher. Since in empirical studies realized stock returns are used as proxy for expected stock returns, the relevant CAPM to test is the one formulated by a PI investor. From the perspective of this hypothetical investor, a higher "full-information" beta is associated with a higher "full-information" expected security return.

In the traditional CAPM, which considers that investors are homogeneously informed, this beta is obtained by regressing a stock's return onto a value-weighted stock index serving as proxy for the market portfolio. The systematic risk premium of the security is the product of the market beta and the expected return of the market portfolio in excess of the risk-free rate. In our study, we estimate the "full-information" beta, which may be considered as a measure of the "real" systematic risk of securities, and study its link with future realized returns in order to test the "full-information" CAPM. According to REE models, the "real beta" is the ratio of the covariance between the realized stock return of a security and the portfolio conditional on all the investors' information (publicly available but also private information), divided by the variance of this portfolio's return. This portfolio is the optimal informed investors' portfolio.

We test the CAPM with two different proxies for the market portfolio: (i) a value-weighted stock index, which is the traditional benchmark used in existing empirical tests of the CAPM; (ii) the optimal informed investors' portfolio, which is the relevant benchmark to use under the assumption that realized returns are a proxy for expected returns. We find that beta is higher when the traditional market index is used. The Fama-MacBeth procedure does not support the traditional CAPM, as in previous studies, and rejects the existence of other factors explaining systematically stock returns. With the informed portfolio used as a benchmark, the results are better. Conditioning on information brings us closer to a positive relation between the market beta and average returns as predicted by the Sharpe–Lintner–Black model.

While our results seem promising, they are not strong enough to support the conditional CAPM. Several avenues of research are possible to improve our approach. Regarding the strategy for building the PI's investor's portfolio, it seems necessary to pursue the research on finding a more relevant proxy for noise in order to extract the available information. In most REE models, noise is represented by supply uncertainty. We have used the relative volume (expressed in euros) of common stocks traded on the market as a proxy for supply uncertainty, but other proxies for supply may generate better results.

Regarding the CAPM test, we find that conditioning on information does not allow the CAPM to explain asset-pricing anomalies. According to Jagannathan and Wang (1996), the effect of these anomalies may be reduced by incorporating time variation in risk premia and the return on human capital as part of the returns on aggregate wealth. Lettau and Ludvigson (2001) affirm: "*Fama-French factors are mimicking portfolios for risk factors associated with time variation in risk premia. Once the (C)CAPM is modified to account for such time variation, it performs about as well as the Fama-French model in explaining the cross-sectional variation in average returns*".

Our preliminary results do not contribute only to the academic field; they may also be relevant for practitioners. One possible future contribution may be to better assess the performance obtained by actively managed equity mutual funds. Such mutual funds generate performance thanks to information acquisition activities related to their expertise on specific market segments. The assumption of asymmetric information is realistic in this field. The market portfolio being irrelevant as a benchmark on markets with asymmetric information, alphas estimated by including the market factor of risk (such as the Jensen's alpha) or other widely used factors (as in the Carhart's model for example) may be irrelevant. Probably, the PI investor's portfolio may be relevant to use as a benchmark in such performance models. Using such a benchmark may provide a different picture of actively managed mutual funds, existing studies in this field showing that their performance is insufficient, which seems paradoxical in light of the importance of active management. If mutual funds would consistently underperform the market, their very existence would be crippled, while in practice the deposits size has dramatically increased over the last decades[15].

Finally, there is still room for improvements in the testing procedure. The Fama-MacBeth procedure has the advantage of its simplicity. To correct the errors-in-variable bias, we use

[15] Deposits in undertakings for collective investment (UCI) governed by French law has for example increased from 888 billion of euros in 2001 (1,473 with mandates and UCI organized under foreign laws) to 1,683 in 2015 (respectively 3,591). Source: AMF.

Litzenberger and Ramaswamy (1979), but there are many other possibilities (see for example Shanken, 1992; Hansen and Jagannathan, 1997; Jagannathan and Wang, 1998; Kan, Robotti, and Shanken, 2013). More importantly, these corrections apply to time-invariant coefficients, within the FM method, which implies that risk premiums are constant, while there is strong evidence that the price of risk varies over time (Campbell and Shiller, 1988; Cochrane, 2011). We leave for a forthcoming version of this paper the use of Adrian, Crump, and Moench (2015)'s dynamic version of the Fama-MacBeth estimator, as well as including the return on human capital in accordance with Jagannathan and Wang (1996).

Fama and French (1992) wrote: "*Resuscitation of the SLB model requires that a better proxy for the market portfolio (a) overturns our evidence that the simple relation between beta and average stock returns is flat and (b) leaves beta as the only variable relevant for explaining average returns*". We hope that we will be able to take up this challenge by extending this work.

Appendix A. Derivation of the CAPM Conditional on PI Investors' Information

Expressing returns as a ratio rather than as price differences:

$$\frac{E_I(\tilde{F}_i - \tilde{P}_i^0) - r_f E_I(\tilde{P}^0)}{\tilde{P}_i^0} = \frac{\rho_a^{-1} Cov_I(\tilde{F}_i - R\tilde{P}_i^0, \tilde{F}_{opt.I} - R\tilde{P}_{opt.I}^0)}{\tilde{P}_i^0}$$

$$\Leftrightarrow \quad E_I(\frac{\tilde{F}_i - \tilde{P}_i^0}{\tilde{P}_i^0}) - r_f = \rho_a^{-1} Cov_I(\frac{\tilde{F}_i - \tilde{P}_i^0 - (R-1)\tilde{P}_i^0}{\tilde{P}_i^0}, \tilde{F}_{opt.I} - R\tilde{P}_{opt.I}^0)$$

$$\Leftrightarrow \quad E_I(\tilde{r}_i) - r_f = \rho_a^{-1} Cov_I(\frac{\tilde{F}_i - \tilde{P}_i^0}{\tilde{P}_i^0} - r_f, \frac{\tilde{F}_{opt.I} - \tilde{P}_{opt.I}^0}{\tilde{P}_{opt.I}^0} - r_f)\tilde{P}_{opt.I}^0)$$

$$\Leftrightarrow \quad E_I(\tilde{r}_i) - r_f = \rho_a^{-1} \tilde{P}_{opt.I}^0 Cov_I(\tilde{r}_i, \tilde{r}_{opt.I}) \tag{A1}$$

Then multiplying (A1) by D_I:

$$(E_I(\tilde{F}_i - \tilde{P}_i^0) - r_f E_I(\tilde{P}^0))D_I = \rho_a^{-1} Cov_I(\tilde{F}_i - \tilde{P}_i^0, \tilde{F}_{opt.I} - \tilde{P}_{opt.I}^0)D_I$$

$$\Leftrightarrow \quad E_I(\tilde{F}_{opt.I} - \tilde{P}_{opt.I}^0) - r_f E_I(\tilde{P}_{opt.I}^0) = \rho_a^{-1} Cov_I((\tilde{F}_i - \tilde{P}_i^0)D_I, \tilde{F}_{opt.I} - \tilde{P}_{opt.I}^0)$$

$$\Leftrightarrow \quad E_I(\tilde{F}_{opt.I} - \tilde{P}_{opt.I}^0) - r_f E_I(\tilde{P}_{opt.I}^0) = \rho_a^{-1} Cov_I(\tilde{F}_{opt.I} - \tilde{P}_{opt.I}^0, \tilde{F}_{opt.I} - \tilde{P}_{opt.I}^0)$$

$$\Leftrightarrow \quad E_I(\tilde{F}_{opt.I}) - \tilde{P}_{opt.I}^0) - r_f E_I(\tilde{P}_{opt.I}^0) = \rho_a^{-1} Var_I(\tilde{F}_{opt.I} - \tilde{P}_{opt.I}^0)$$

$$\Leftrightarrow \quad \tilde{P}_{opt.I}^0 \left(E_I \left(\frac{\tilde{F}_{opt.I} - \tilde{P}_{opt.I}^0}{\tilde{P}_{opt.I}^0} \right) - r_f E_I \left(\frac{\tilde{P}_{opt.I}^0}{\tilde{P}_{opt.I}^0} \right) \right) = \rho_a^{-1} Var_I \left(\frac{\tilde{F}_{opt.I} - \tilde{P}_{opt.I}^0}{\tilde{P}_{opt.I}^0} \right) (\tilde{P}_{opt.I}^0)^2$$

$$\Leftrightarrow \quad E_I(\tilde{r}_{opt.I}) - r_f = \rho_a^{-1} Var_I(\tilde{r}_{opt.I})\tilde{P}_{opt.I} \tag{A2}$$

We divide (A1) by (A2) and obtain the asset-pricing model from the PI investor's perspective:

$$E_I(\tilde{r}_i) - r_f = \frac{\rho_a^{-1} \tilde{P}^0_{opt.I} Cov_I(\tilde{r}_i, \tilde{r}_{opt.I})}{\rho_a^{-1} Var_I(\tilde{r}_{opt.I}) \tilde{P}_{opt.I}} (E_I(\tilde{r}_{opt.I}) - r_f)$$

$$\Leftrightarrow \quad E_I(\tilde{r}_i) - r_f = \beta_{I,i}(E_I(\tilde{r}_{opt.I}) - r_f) \qquad \text{(A3)}$$

where $\beta_{I,i}$:

$$\beta_{I,i} = \frac{Cov_I(\tilde{r}_i, \tilde{r}_{opt.I})}{Var_I(\tilde{r}_{opt.I})}$$

Appendix B. Fama-French Industrial Classification

Description of the five industrial portfolios:

1. **Cnsmr:** Consumer Durables, NonDurables, Wholesale, Retail, and Some Services (Laundries, Repair Shops)
2. **Manuf:** Manufacturing, Energy, and Utilities
3. **HiTec:** Business Equipment, Telephone and Television Transmission
4. **Hlth:** Healthcare, Medical Equipment, and Drugs
5. **Other:** Mines, Construction, Construction Materials, Transportation, Hotels, Business Services, Entertainment, Finance

Please consult Ken French's website for the distribution of the SIC codes in each portfolio: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/det_5_ind_port.html

Appendix C. Solving the Issue of Non-Full Rank Model

Let's consider a model of the form $Y = \beta \cdot X$, with Y a $(T \times 1)$ vector, X a $(T \times n)$ matrix, and β a $(n \times 1)$ vector, T being the sample size, and n the number of explanatory variables.

In the case of the optimal uninformed investors' portfolio, we project the return of a Fama-French portfolio onto the (relative) prices of the six FF portfolios. We have thus $n = 6$ and $T = 60$ months. If we include an intercept while the sum of the relative prices equals 1:

$$X = \begin{bmatrix} 1 & P^{t-60}_{FF1} & \cdots & P^{t-60}_{FF6} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & P^{t-1}_{FF1} & \cdots & P^{t-1}_{FF6} \end{bmatrix}$$

where $X_{i,1} = X_{i,2} + \ldots + X_{i,7}$, then $X'X$ is singular, i.e. $det(X'X) = 0$, while an OLS estimation requires $X'X$ invertible ($\hat{\beta} = (X'X)^{-1}X'Y$). Consequently, the model is not full rank (rank 6 instead of 7) and we have to set a parameter to 0 to find least squares solutions (the variables being a linear combination of other variables).

For the uninformed portfolio, we can easily address this issue by removing the intercept. However, it becomes more complicated for the optimal informed investors' portfolio which is obtained by regressing the return of a FF portfolio onto the (relative) prices and (relative) volumes of the six FF portfolios. Two options:

- Not calculating volumes relatively, but coefficients would be very small and there is no theoretical justification to divide volumes by 10^{11} only to put them on a scale;
- Removing one volume from the regression.

We opted for this second option. Even if in this way we lose a bit of information, it is more relevant to think in terms of traded portfolios rather than in terms of traded holdings, and calculating relative portfolios seem to us the best method to achieve this.

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Table I
Volatilities over the period 8/1996 – 12/2014

The monthly ex ante volatility of the index, V_{index}^{t-1}, is computed as the standard deviation of the residuals from the projection of the index return onto the six FF portfolio prices, i.e. the root mean squared difference between its return and that predicted by the regression. The monthly ex ante volatility of the uninformed and informed portfolios, $\sigma(q^{t\prime}r^t|P^{t-1})$ and $\sigma(q^{t\prime}r^t|P^{t-1}, Z^t)$, are computed as follows, $\sqrt{q \times \Sigma \times q'}$ where Σ is the variance-covariance matrix, and are matched with the ex ante volatility of the index.
The ex post volatilities are traditionally computed as the standard deviation of the realized returns, i.e. the root mean squared differences between each realized return and the average realized return. The conservative method of Biais et al. (2010) computes it as the root mean squared differences between each realized return and the average expected return.

Volatility (in %)	Method	Index	Uninformed	Informed
Ex ante	Mean of the monthly volatilities	4.65	4.65	4.65
Ex post	Traditional	5.06	6.08	6.69
	Conservative	5.08	7.17	8.04

Table II
Descriptive statistics, 1/1991 to 12/2014

6 FF portfolios

	Portfolio	Mean (in %)	Standard Deviation (in %)	Kurtosis	Skewness
B/L	Big, Low Value	0.823	0.253	0.7	−0.25
B/M	Big, Medium Value	1.010	0.274	1.1	−0.46
B/H	Big, High Value	1.017	0.406	1.3	−0.26
S/L	Small, Low Value	0.597	0.309	14.7	1.89
S/M	Small, Medium Value	0.706	0.174	2.0	−0.40
S/H	Small, High Value	1.226	0.166	3.2	0.06

	SMB	HML	Market
Mean return (in %)	−0.11	0.41	0.84

Table III

Performance of the uninformed and informed portfolios, 8/1996 to 12/2014

Jensen's alpha obtained from the (i) projection of the optimal uninformed and informed portfolios returns onto the market return: $r_p^t - r_f^t = \alpha_p + \beta_p \cdot (r_m^t - r_f^t) + \epsilon^t$; (ii) addition of the Fama-French (1993) zero-investment portfolio SMB (Small Minus Big) and HML (High Minus Low): $r_p^t - r_f^t = \alpha_p + \beta_p \cdot (r_m^t - r_f^t) + s_p \cdot SMB^t + h_p \cdot HML^t + \epsilon_p^t$; (iii) inclusion of the Carhart (1997) zero investment portfolio UMD (Up Minus Down): $r_p^t - r_f^t = \alpha_p + \beta_p \cdot (r_m^t - r_f^t) + s_p \cdot SMB^t + h_p \cdot HML^t + u_p \cdot UMD^t + \epsilon_p^t$. The t-statistics in brackets test whether the coefficients are different from zero. The Jensen's alpha measures the performance above the return which would be justified by the systematic risk. The Treynor ratio adjusts for the market risk the returns gained in excess of a risk-less investment, $T_p = \frac{r_p - r_f}{\beta_p}$, with r_p the time-series average of the portfolio return, r_f the time-series average of the risk-free rate and β_p the beta of the portfolio.

	Alpha			Treynor ratio
	CAPM	3-factor	4-factor	
Uninformed	0.0055	0.0034	0.0047	0.0149
	(2.48)	(1.62)	(2.11)	
Informed	0.0074	0.0055	0.0065	0.0172
	(3.56)	(2.89)	(3.10)	

Table IV
Average slope coefficients of the portfolios, 8/1996 to 12/2014

Average slope coefficients from the following regressions: (i) $r^t_{FFj} = \sum_{j=1}^{6} \beta_j \cdot P^{t-1}_{FFj} + \varepsilon^t_i$ for the optimal uninformed investors' portfolio; (ii) $r^t_{FFj} = \sum_{j=1}^{6} \beta_j \cdot P^{t-1}_{FFj} + \sum_{j=1}^{6} \beta_{j+6} \cdot Z^t_{FFj} + \varepsilon^t_i$ for the optimal informed investors' portfolio. In parentheses is reported the average standard deviation of the estimated slope coefficients. The slope coefficients and their standard deviation are first averaged across the six regressions, and then time averaged.

Panel A: Optimal Uninformed Investors' Portfolio

	S/L	S/M	S/H	B/L	B/M	B/H
β with P^{t-1}	−12.10	−0.25	2.21	−0.03	0.25	−0.36
	(32.63)	(19.50)	(6.28)	(0.65)	(0.44)	(0.57)

Panel B: Optimal Informed Investors' Portfolio

	S/L	S/M	S/H	B/L	B/M	B/H
β with P^{t-1}	11.42	−11.20	7.63	0.09	0.10	−0.46
	(38.59)	(22.05)	(7.87)	(0.76)	(0.50)	(0.67)
β with Z^{t-1}	−1.40	−0.49	0.47	0.01	0.00	0.00
	(1.90)	(1.16)	(1.29)	(0.01)	(0.02)	(0.05)

Table V
Descriptive statistics

Panel A: Overview

	Subperiod	Duration
1. Portfolio formation	1996–2000	48 months
2. Estimation	2000–2005	60 months
3. Testing	2005–2009	48 months

Yearly periods are from the end of August t to the end of July $t+1$.

Panel B: Number of stocks allocated to portfolios

Portfolio	1	2–19	20	Total
Nb of stocks	17	12	18	251

The middle 18 portfolios each has $int(N/20)$ stocks, and the first and last portfolios each receives:

$$int\left(\frac{N}{20}\right)+\frac{1}{2}\left[N-20int\left(\frac{N}{20}\right)\right]$$

with N the total number of securities to be allocated to portfolio and $int(N/20)$ be the largest integer equal to or less than $N/20$. When N is odd like here, the last portfolio (highest $\hat{\beta}$) gets an additional security.

Panel C: Average betas

		Average β_i		
Period	Updates	Informed	Uninformed	Index
1996–1999		0.109	0.110	0.654
2000–2005		0.149	0.257	0.750
	2006	0.167	0.257	0.777
2005–2008	2007	0.163	0.246	0.773
	2008	0.265	0.349	0.812
Time-average		0.171	0.244	0.753

Individual betas are computed for each security against: (i) the optimal informed investors' portfolio; (ii) the optimal uninformed investors' portfolio; (iii) and the value-weighted stock index. They are then averaged across securities. 1. We calculate pre-ranking firms' betas with time-series from 8/1996 to 7/2000. 2. We calculate post-ranking firms' betas with time-series from 8/2000 to 7/2005. 3. Firms' betas are yearly updated with time-series from 8/2000 to 7/2006, from 8/2000 to 7/2007 and from 8/2000 to 7/2008

Table VI
Test of the CAPM using Fama and MacBeth (1973)

(i) 20 portfolios are formed according to individual security betas, estimated with a times-series from 30/08/1996 to 31/07/2000.
(ii) Portfolio betas and portfolio non-beta risk are calculated by equal-weighting respectively individual security betas and standard deviations of the residual returns for individual securities, estimated with time-series from 31/08/2000 to 29/07/2005. Firms' betas and the standard deviation of the residuals are then recalculated annually with time-series from 31/08/2000 to 31/07/2006, 31/08/2000 to 31/07/2007, and from 31/08/2000 to 31/07/2008.
(iii) Cross-sectional OLS regressions are run each month from 31/08/2005 to 31/07/2009, adjusting monthly portfolio betas and portfolio non-beta risk by removing delisted firms or having missing data.

In the last column are reported the t-statistics for each coefficient estimate, corrected with Litzenberger and Ramaswamy (1979) methodology.

Panel A

Value-weighted index

	$\bar{\gamma}$	$s(\gamma)$	$t(\gamma)$
γ_0	−0.049	1.021	−0.33
γ_1	0.084	1.434	0.41
γ_2	−0.117	1.294	−0.63
γ_3	0.010	1.159	0.06

$R^2 = 0.095$

Optimal informed investors' portfolio

	$\bar{\gamma}$	$s(\gamma)$	$t(\gamma)$
γ_0	−0.037	1.227	−0.21
γ_1	0.222	0.997	1.54
γ_2	−0.137	1.068	−0.89
γ_3	−0.473	1.220	−2.69

$R^2 = 0.054$

Panel B

Optimal uninformed investors' portfolio

	$\bar{\gamma}$	$s(\gamma)$	$t(\gamma)$
γ_0	0.037	1.046	0.25
γ_1	−0.114	0.947	−0.84
γ_2	0.107	0.940	0.79
γ_3	−0.075	1.140	−0.46

$R^2 = 0.001$

Table VII
Comparing value- and equal-weighting

(i) 20 portfolios are formed according to individual security betas, estimated with a times-series from 30/08/1996 to 31/07/2000.
(ii) Portfolio betas and portfolio non-beta risk are calculated by equal-weighting respectively individual security betas and standard deviations of the residual returns for individual securities, estimated with time-series from 31/08/2000 to 29/07/2005. Firms' betas and the standard deviation of the residuals are then recalculated annually with time-series from 31/08/2000 to 31/07/2006, 31/08/2000 to 31/07/2007, and from 31/08/2000 to 31/07/2008.
(iii) Cross-sectional OLS regressions are run each month from 31/08/2005 to 31/07/2009, adjusting monthly portfolio betas and portfolio non-beta risk by removing delisted firms or having missing data.

In the last column are reported the t-statistics for each coefficient estimate, corrected with Litzenberger and Ramaswamy (1979) methodology.

Value-weighted index

	$\bar{\gamma}$	$s(\gamma)$	$t(\gamma)$
γ_0	−0.049	1.021	−0.33
γ_1	0.084	1.434	0.41
γ_2	−0.117	1.294	−0.63
γ_3	0.010	1.159	0.06

$R^2 = 0.095$

Equally-weighted index

	$\bar{\gamma}$	$s(\gamma)$	$t(\gamma)$
γ_0	0.175	1.301	0.93
γ_1	−0.084	1.485	−0.39
γ_2	0.057	1.176	0.34
γ_3	−0.149	1.185	−0.87

$R^2 = 0.157$

Table VIII
Summary statistics for 100 FF portfolios sorted on size and beta, 7/1999 to 12/2014

Panel A: Time-Series Averages of Returns (in %)

	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9	β_{10}
S1	0.94	2.02	1.52	1.61	2.35	1.84	2.16	1.55	0.83	0.82
S2	0.66	0.18	1.35	1.01	1.46	0.61	0.84	0.21	0.78	0.92
S3	0.45	0.50	0.26	0.83	1.51	0.16	0.98	0.77	1.39	0.31
S4	0.46	1.01	1.31	1.14	0.27	0.43	0.45	0.74	0.44	0.45
S5	0.79	0.99	1.21	0.28	0.52	0.60	0.72	0.75	0.34	0.35
S6	0.67	0.94	1.02	1.22	0.82	1.04	1.02	1.09	0.76	0.71
S7	1.18	0.69	1.44	0.98	0.80	0.82	1.79	0.87	0.73	0.28
S8	1.04	1.00	1.24	1.24	1.01	1.01	0.93	1.07	0.58	0.32
S9	1.29	1.04	1.15	0.59	1.00	1.43	1.45	0.61	0.56	0.62
S10	1.27	0.50	1.04	0.75	0.62	0.99	0.66	0.61	0.30	0.63

Panel B: The Estimated Betas

	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9	β_{10}
S1	0.63	0.17	0.39	0.42	0.54	0.68	0.58	0.95	0.94	1.22
S2	0.35	0.38	0.21	0.44	0.46	0.75	1.02	0.76	1.08	1.26
S3	0.30	0.41	0.33	0.56	0.75	0.68	0.84	0.88	1.26	1.44
S4	0.33	0.37	0.42	0.49	0.63	0.55	0.73	0.90	1.15	1.51
S5	0.60	0.26	0.54	0.67	0.66	0.64	0.79	0.86	1.32	1.79
S6	0.43	0.46	0.48	0.42	0.53	0.81	0.76	0.95	1.19	1.60
S7	0.41	0.36	0.46	0.69	0.59	0.78	0.82	1.06	1.34	1.46
S8	0.33	0.40	0.66	0.67	0.84	0.75	1.00	1.21	1.62	1.81
S9	0.45	0.59	0.68	0.63	0.81	0.83	1.09	0.95	1.40	1.66
S10	0.52	0.78	0.77	0.93	0.87	1.21	1.26	1.40	1.47	1.81

Panel C: The Time-Series Averages of Size (log million €)

	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9	β_{10}
S1	1.46	1.63	1.63	1.67	1.69	1.72	1.75	1.67	1.64	1.60
S2	2.54	2.58	2.62	2.63	2.66	2.59	2.60	2.63	2.69	2.64
S3	3.29	3.27	3.21	3.29	3.24	3.25	3.23	3.27	3.23	3.24
S4	3.76	3.79	3.83	3.85	3.75	3.82	3.84	3.78	3.76	3.79
S5	4.32	4.35	4.38	4.32	4.29	4.35	4.30	4.30	4.30	4.30
S6	4.88	4.93	4.90	4.91	4.88	4.92	4.95	4.88	4.87	4.88
S7	5.49	5.51	5.52	5.53	5.52	5.50	5.53	5.53	5.51	5.50
S8	6.30	6.30	6.35	6.33	6.30	6.31	6.27	6.28	6.29	6.22
S9	7.31	7.39	7.24	7.46	7.48	7.48	7.55	7.45	7.50	7.51
S10	9.31	9.71	9.48	9.33	9.29	9.41	9.47	9.31	9.30	9.40

The size of a stock is defined as the logarithm of the market value of the asset.

Table IX

Fama-MacBeth two-step procedure applied to the CAPM using 100 FF portfolios, Monthly returns, 7/1999 to 12/2014

Fama and MacBeth (1973) two-stage test procedure is applied to monthly returns from July 1999 to December 2014 for the 100 Fama-French portfolios sorted by size and beta:

(i) Portfolio returns are regressed against variables hypothesized to explain expected returns:

$$R_i^t = \beta_{0,i} + \beta_{1,i} \cdot MKT^t + \varepsilon_i^t$$

The return on a value-weighted stock index is used as proxy for the market return in the left table, compared with the return on the optimal informed investors' portfolio in the right table. These time-series regressions, from 30/08/1996 to 31/12/2014, deliver estimates of the portfolio beta.

(ii) Each month, from 30/08/1996 to 31/12/2014, the cross-section of returns on portfolios is regressed on these factor exposures:

$$R_i^t = \lambda_{0,i} + \lambda_1^t \cdot \hat{\beta}_{1,i} + \eta_i^t$$

Coefficients are then weighted by the inverse of their standard error, and time series averaged to obtain the risk premium of each factor.

In the last column are reported the t-statistics for each coefficient estimate, corrected with Litzenberger and Ramaswamy (1979) methodology.

Value-weighted index

	$\bar{\gamma}$	$s(\gamma)$	$t(\gamma)$
γ_0	0.836	2.385	4.78
γ_1	−0.489	3.884	−1.72

$R^2 = 0.118$

Optimal informed investors' portfolio

	$\bar{\gamma}$	$s(\gamma)$	$t(\gamma)$
γ_0	0.642	1.821	4.81
γ_1	−0.315	2.698	−1.59

$R^2 = 0.019$

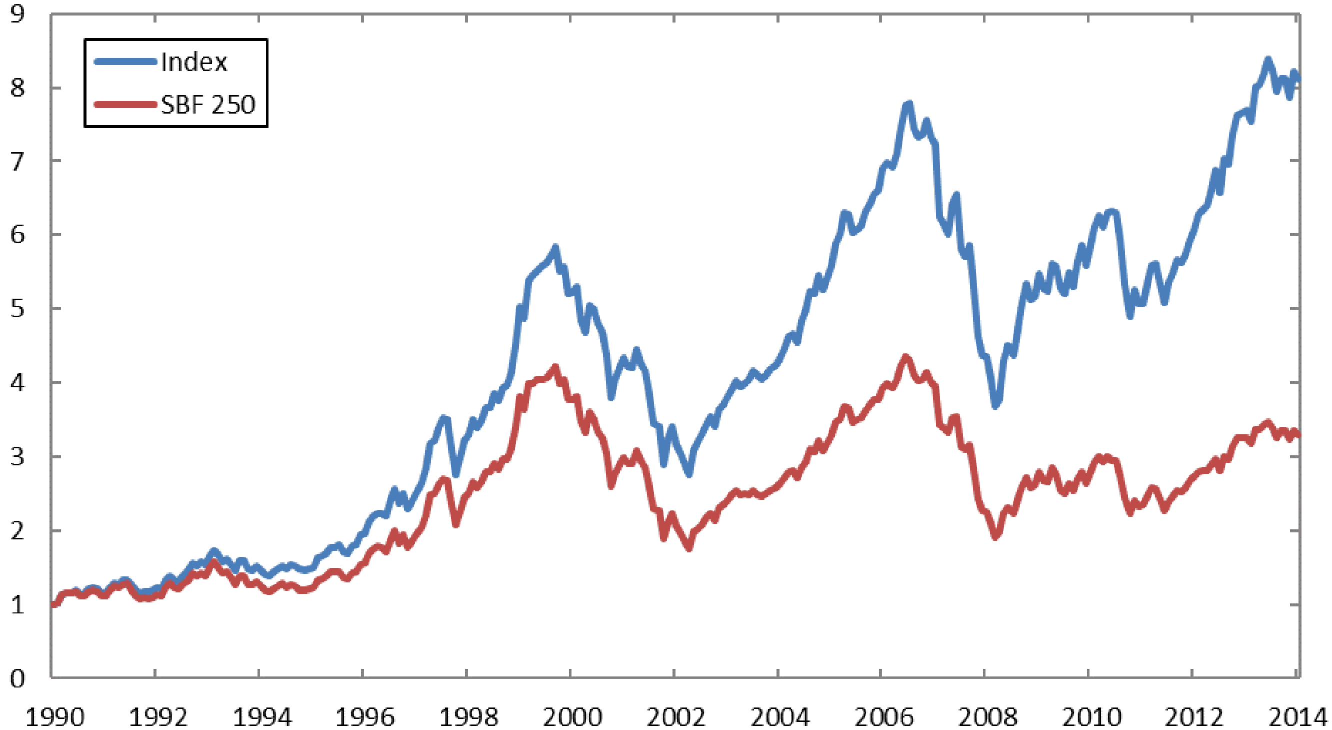

Figure 1. Comparison of our index with the SBF 250. Evolution of the cumulated wealth for 1€ invested, 12/1990 to 12/2014. The SBF 250 was launched on December 28, 1990 with a base of 1,000 points and replaced on March 21, 2011 by the CAC All-Tradable.

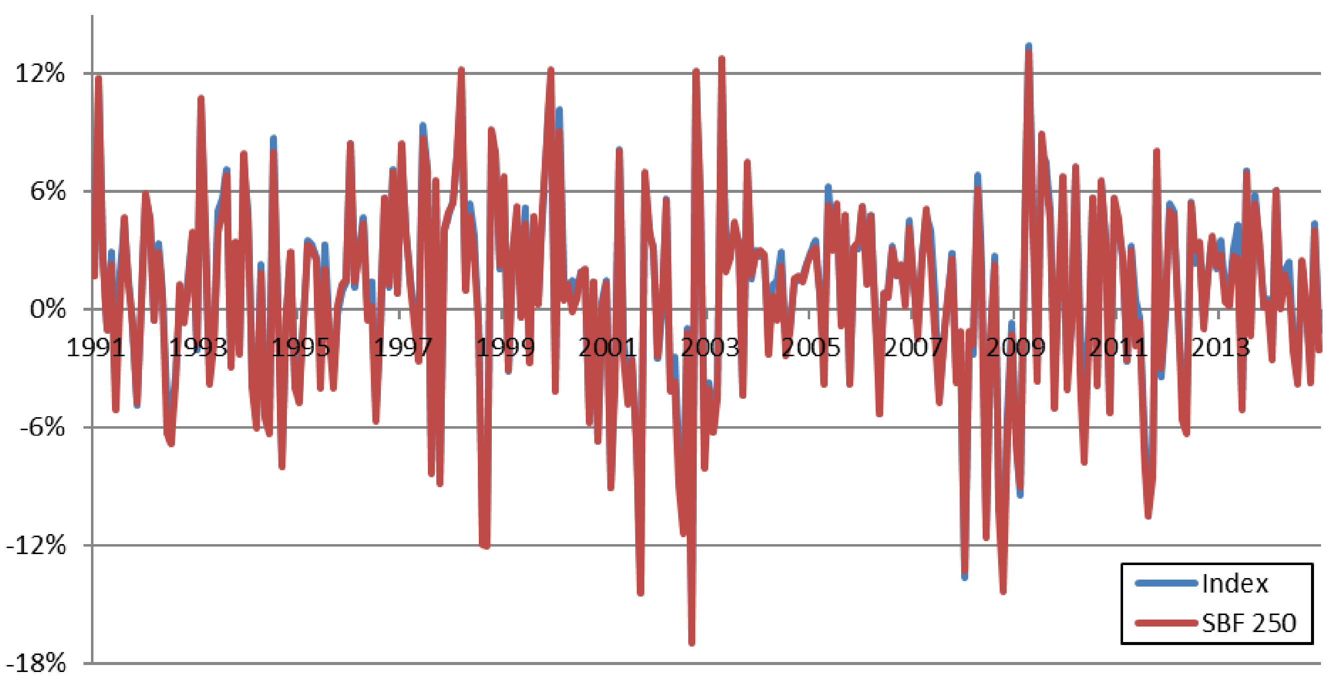

Figure 2. Comparison of our index with the SBF 250. Monthly returns, 1/1991 to 12/2014, dividends are reinvested.

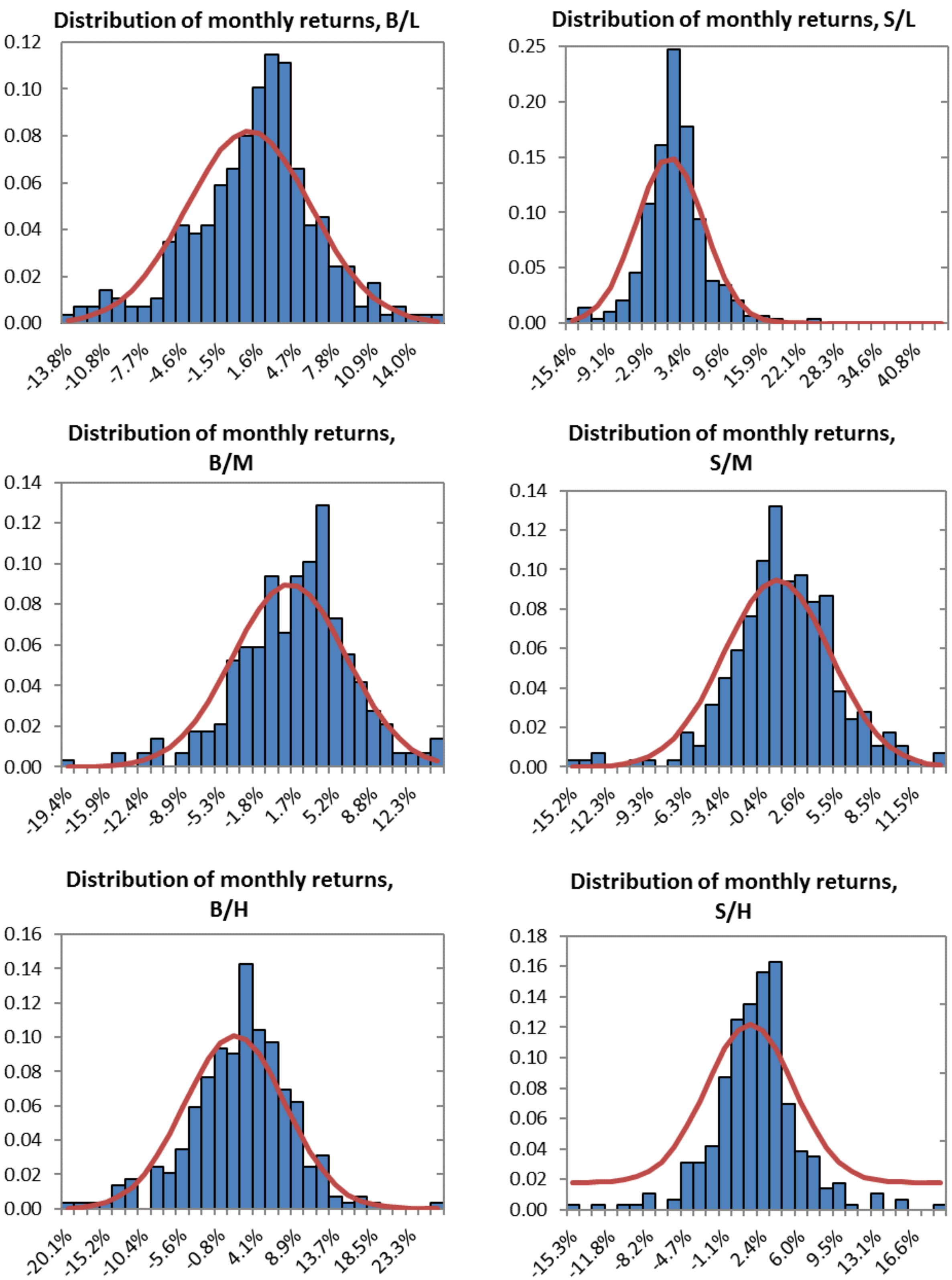

Figure 3. Distribution of monthly returns. 6 FF portfolios, 1/1991 to 12/2014.

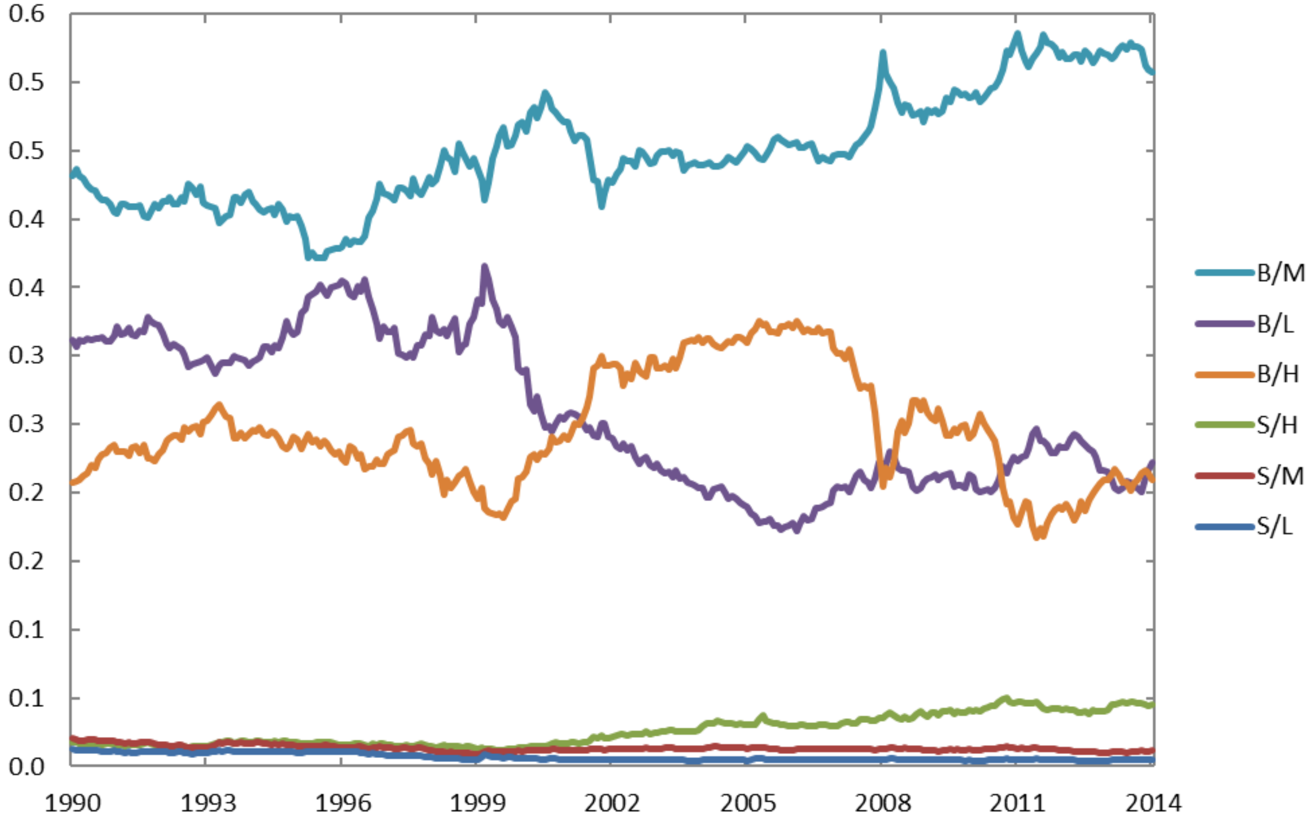

Figure 4. Prices of the six FF portfolios. Evolution of the relative prices of the six FF portfolios, 12/1990 to 12/2014. They are computed as weights in a buy-and-hold portfolio, initialized by value-weighting on December 31, 1990. Dividends are reinvested in the component portfolios that generated them.

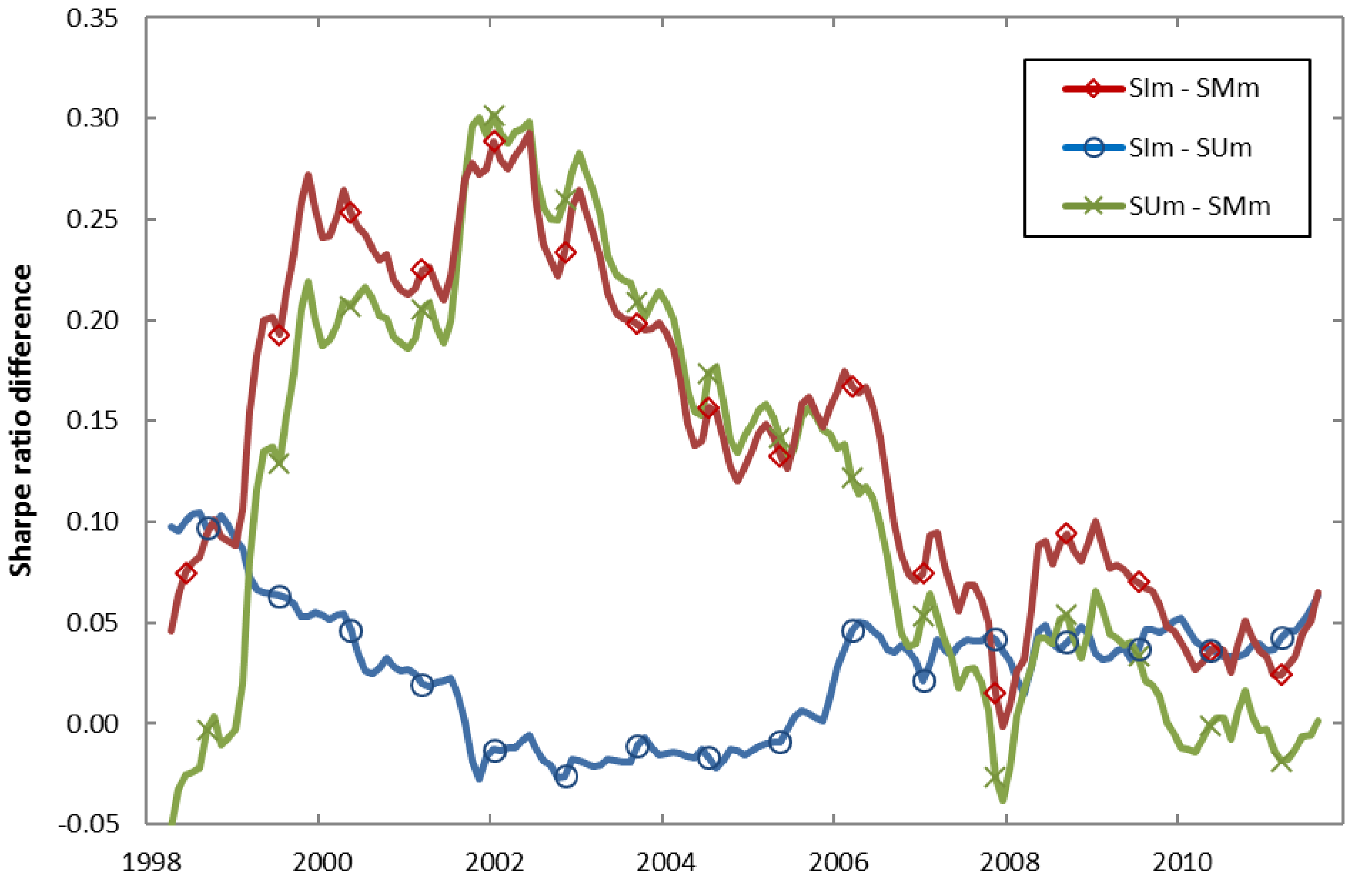

Figure 5. Performance results. Evolution of the difference between the Sharpe ratios, 2/1999 - 6/2012. (i) Difference between the Sharpe ratio of the informed portfolio (SIm) and the Sharpe ratio of the market portfolio (SMm); (ii) Difference between the Sharpe ratio of the informed portfolio and the Sharpe ratio of the uninformed portfolio (SUm); (iii) Difference between the Sharpe ratios of the uninformed and market portfolios.

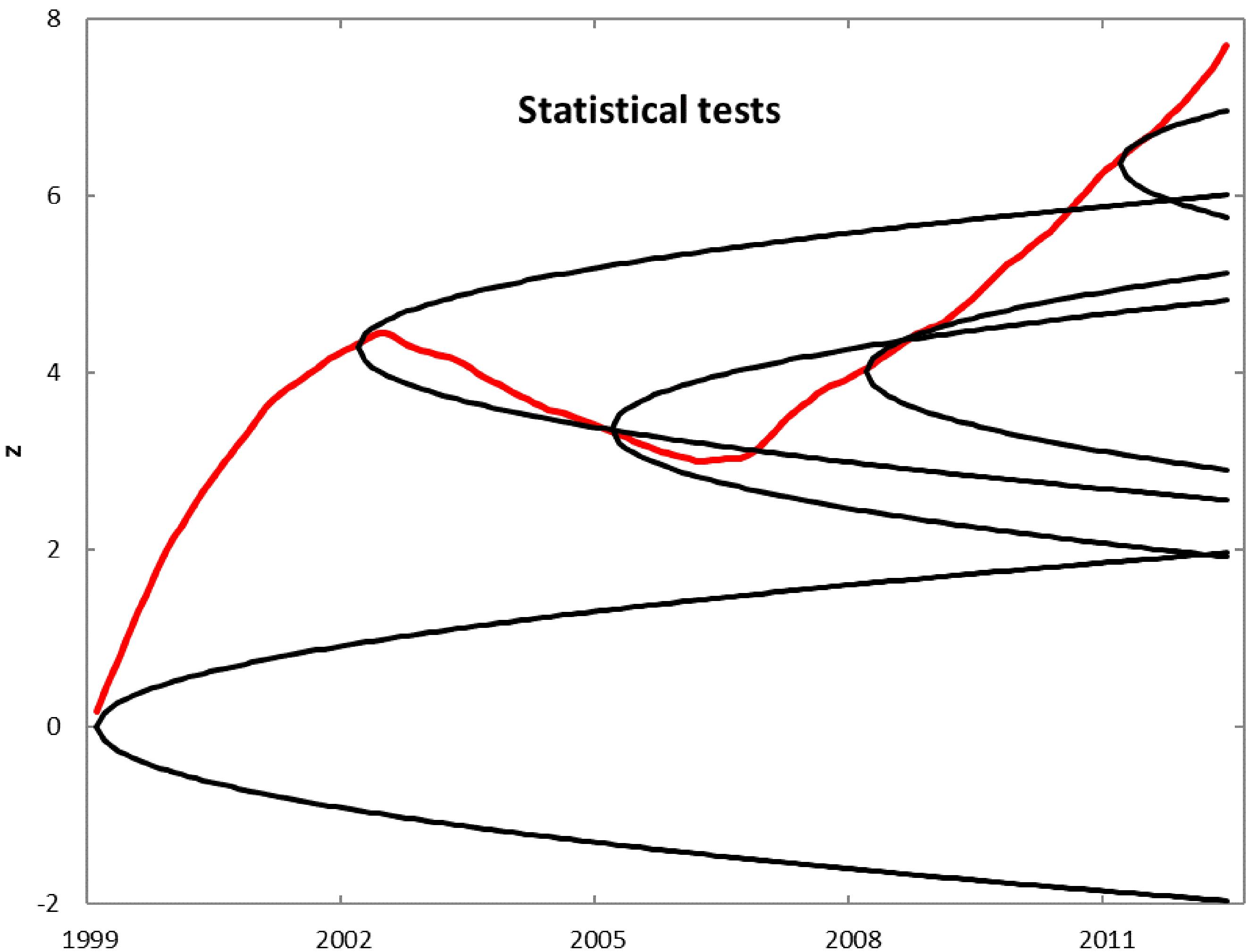

Figure 6. Statistical tests. Evolution of the partial z-statistic of the difference in return between the optimal informed portfolio and the optimal uninformed portfolio. Parabolas represent for each month the regions of acceptance of the null hypothesis with 95% confidence that the Sharpe ratios of the informed and uninformed portfolio are not statistically different. At one date, when the partial z-statistic is outside the two limits of the confidence region, then the difference between the Sharpe ratios is statistically significant at the 0.05 level. The partial z-statistic is plot from 2/1999 to 6/2012, divided in five subperiods, four 36-month length intervals and a last one of 17 months.

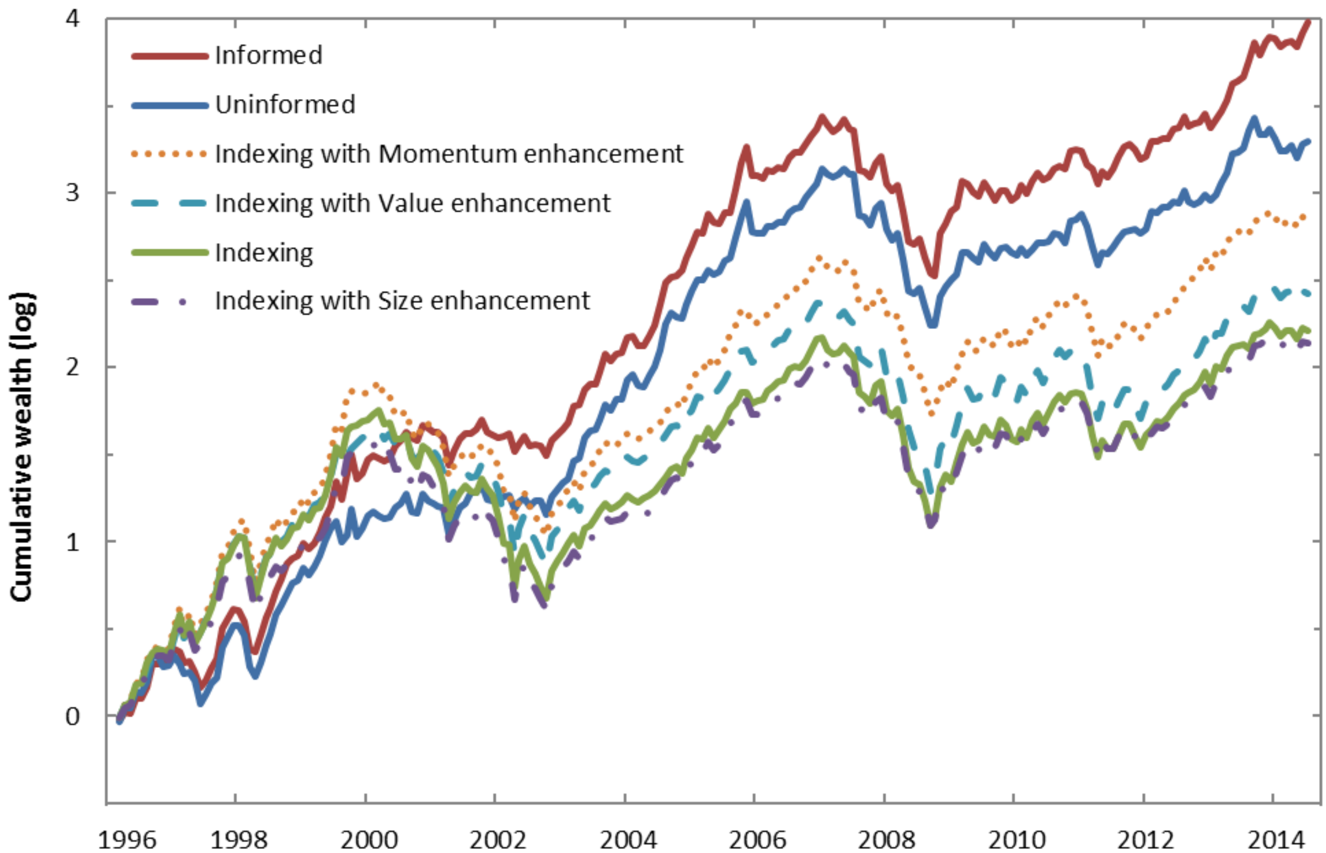

Figure 7. Performance comparison. Log base 2 of the wealth evolution of 1€ invested on July 31, 1996 with monthly returns reinvested. Each time a line increases by 1 unit, the value doubles. All the strategies are determined so that their ex post volatilities are approximately equal: (i) The informed agent's strategy is to constantly invest 47% of his wealth in the optimal informed portfolio and the remaining in the market portfolio; (ii) The strategy of an uninformed agent is to invest 56% in the optimal uninformed portfolio and 44% in the market portfolio; (iii) Indexing with momentum enhancement (95% investment in the index, 5% in the one-month Treasury bill, plus 20% in the zero-investment momentum portfolio which is long position on past winners, i.e. firms with the highest 30 percent eleven-month returns lagged one month, and short on past losers, i.e. firms with the lowest 30 percent eleven-month returns lagged one month); (iv) Indexing with value enhancement (95% investment in the index, 5% in the one-month Treasury bill, plus 25% in the zero-investment value portfolio which go long firms with the highest three decile book-to-market ratio, and short firms with the lowest three decile book-to-market ratio); (v) Indexing with size enhancement (95% investment in the index, 5% in the one-month Treasury bill, plus 20% in the zero-investment size portfolio which shorts firms with market capitalization above the median to buy firms below the median) (vi) Indexing (value-weighted index).

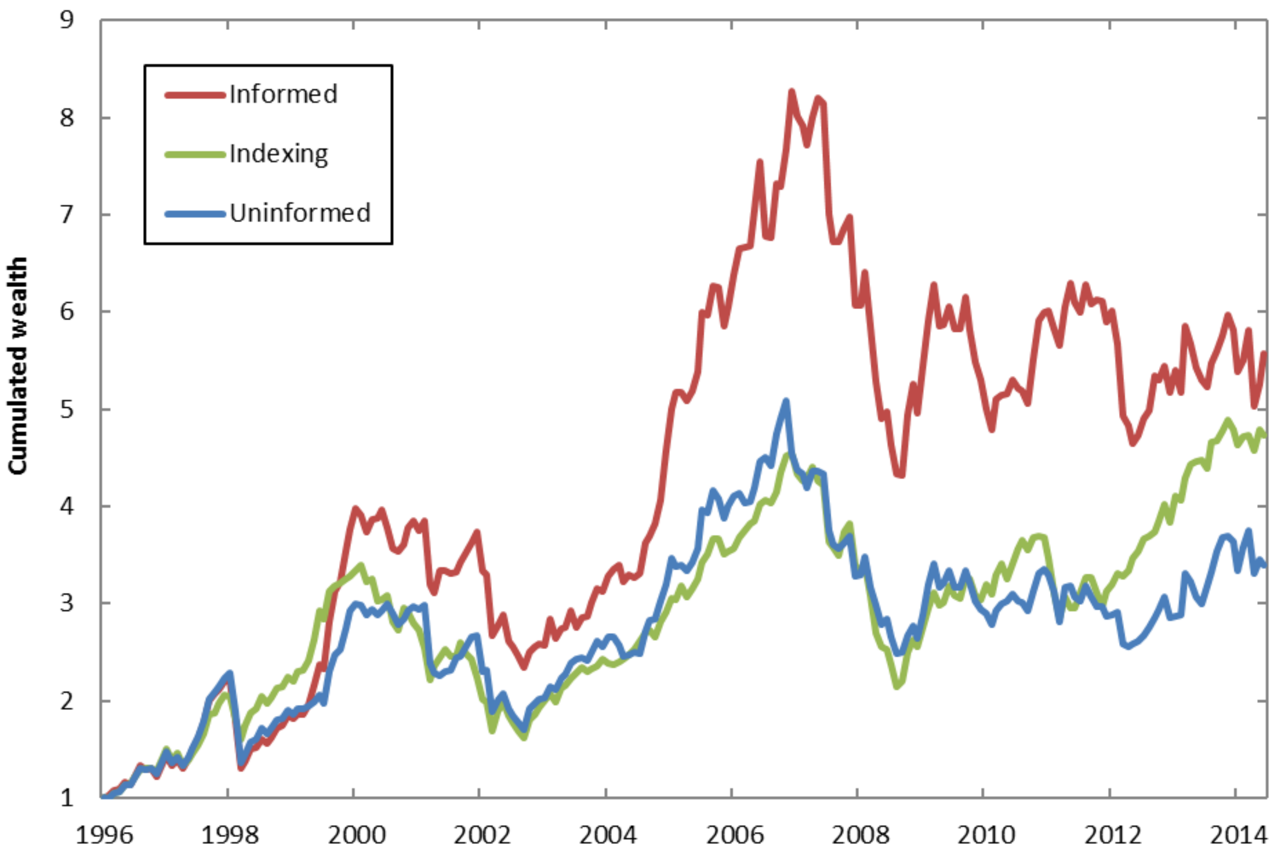

Figure 8. Industry portfolios. Performance comparison with a grouping of stocks according to their industries, 8/1996 to 12/2014. Wealth evolution of 1€ invested on July 31, 1996 with monthly returns reinvested.