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Evgenia Shabalina

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Unité de recherche : Institut de Physique de Rennes – UMR 6251

Rapporteurs avant soutenance :

Emmanuelle Rio
Maître de Conférence, LPS, Université Paris Sud

Simon Cox
Professor, Aberystwyth University

Composition du Jury :

Hamid Kellay
Professeur des universités, Université de Bordeaux

Benoit Scheid
Professeur des universités, Université Libre de Bruxelles

Anne-Laure Biance
Chargée de recherche, CNRS

Directeur de thèse
Isabelle Cantat

Co-directeur de thèse
Arnaud Saint-Jalmes

Professeur des universités, Université de Rennes 1

Directeur de Recherche CNRS, Directeur-Adjoint IPR
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Introduction

Liquid foams are concentrated dispersions of gas bubbles in a surfactant solution and they have specific properties, which are broadly used in the industry: food and cosmetics production, as drilling fluids in oil production, as fire fighting agents for polar solvent and oil fires. Foams have mechanical properties which can be either solid-like or liquid-like and this fact is the object of much research [30]. They demonstrate quasi-elastic behavior up to finite yield stress or strain, further they flow like shear-thinning viscous fluids. The origin of the elastic response comes from the variation of total film area under deformation.

Foams exhibit different length scales from macroscopic to molecular and physical models of foam behavior can involve interaction between processes at all of these length scales. But relating local measurements to the global rheology of the foams is still tricky. Many techniques to estimate the rheological properties of the films are based on the film’s confinement by solid barriers but in real foams the films are bounded by the Plateau borders which allow liquid transfer from one film to another.

Under foam shearing, the film area increases and it happens both due to the extension of the film or by extraction of a new film from the Plateau border. These facts are expected to contribute significantly to the global dissipation in foam under the shear. The main problem is to try to look for a relationship between the effective viscosity in 3D foam and the physical parameters of the foam. With this aim, we use a few films and try to describe their behaviour under deformation. The processes observed locally should be useful to build a model for the effective viscosity of 3D foams.

The thesis consists of three parts and five chapters as well as the introduction, the conclusion and two appendixes with recently published results of our team.

Part I, called "Basics", includes two chapters. The objective of this part is to introduce basic concepts of soap films and tools used for our experiments.

Chapter 1 explains general information about soap films and provides a literature review of recent studies made about the deformation of soap films and understanding the properties of the Frankel’s films. During experiments we used several optical devices, so one paragraph of the first chapter is dedicated to some optical information.

Chapter 2 explains the techniques to create and to deform a soap film and the optical tools to observe it. Firstly we introduce the set up allowing us to create three soap films connected through a common meniscus (PB). The idea and the design of the set up are developed and fully performed in our team and it may be linked to foam at a millimetric
scale of a few bubbles. One section of Chapter 2 also introduces the chemistry used in the experiments.

Part II contains two chapters explaining experiments carried out with three soap films. First of all, we study a film under stretching and new film extraction from a meniscus in Chapter 3. We build a model to explain the visco-elastic behavior of the stretched film, test the model on our experimental data and compare the results predicted by the model and obtained from experiments. In order to determine the thickness of the extracted film, we use a hyperspectral camera, which was not used previously by other researchers.

In Chapter 4 we describe for the first time a 2D Rayleigh-Taylor instability, which takes place in a soap film under stretching when a thicker film is produced on top of a thin one. We begin with building the linear stability analysis, then, we continue with a set of experiments considering the problem in a large range of experimental control parameters such as the effective gravity and the velocity of stretching. At the end of the chapter, we compare our linear model with the experimental data. Based on this chapter an article has been submitted.

Part III involves only one chapter but it is needed to be well separated from previous parts because it touches the bigger and more complicated problem of a foam sample at millimetric scale. In Chapter 5 we develop and test an original set up of five soap films and two menisci, which has been patented. Using this setup, we provide preliminary results evidencing the dynamical influence of one stretched film on its neighbours. The set up has then been used by A. Bussonnière, while I focused on the instability described in Chapter 4. I am the co-author of the submitted article on Chapter 5.
Part I
Basics
Chapter 1

General information about soap films

1.1 Film structure

In this section, I present thin foam film characteristics at different scales: at the scale of a gas/liquid interface, at the scale of a film and at the scale of a bubble. First of all, we consider a surface tension and the Young-Laplace law, then, I write briefly about surfactants. Second, we discuss film thickness and, finally, we consider films and Plateau borders.

1.1.1 Surface tension

When a liquid molecule is located at the liquid/gas interface, it loses half of its cohesive interactions and it is therefore in a less favorable energy state. An increase in interfacial area by an amount $dS$ induces a change in the energy by an amount

$$dE_{surf} = \gamma dS,$$  \hspace{1cm} (1.1)

where $\gamma$ is the surface tension [18].

The tension $\gamma$ represents an energy per unit area ($J.m^{-2}$) or a force per unit length ($N.m^{-1}$). At liquid/gas interfaces, surface tension results from the greater attraction of liquid molecules to each other (due to cohesion) than to the molecules in the air (due to adhesion).

1.1.2 The Young-Laplace law

Let’s consider the piece $dS$ of the interface around the point M as pictured on Figure 1.1. As we said previously, there is a force per unit length, acting on the perimeter of $dS$, called
the surface tension $\gamma$, which tends to reduce the area of the interface. The pressure difference between two sides of $dS$ is

$$\Delta P = P_j - P_i.$$  \hspace{1cm} (1.2)

$n$ is the normal at $M$ oriented from region $j$ towards $i$. If we imagine a plane parallel to the normal to the surface, we see that this cuts the surface along a line with radius of curvature $R_1$ or if we rotate the plane around the normal, we get $R_2$ (radii are taken in two particular directions, called the principal radii of curvature $R_1$ and $R_2$, which turn out to be perpendicular). By convention, the radius of curvature is considered positive if the center of curvature is in $j$. The pressure increase when we cross the interface from one region to the other is defined by the Young-Laplace law [55], [13], [18]:

$$\Delta P = \gamma \left( \frac{1}{R_1} + \frac{1}{R_2} \right).$$ \hspace{1cm} (1.3)

It indicates that surface tension tends to reduce the curvature of an interface, making it more planar and that it is counterbalanced by a pressure difference, which tends to bend the interface.

Figure 1.1: Scheme of liquid interface. $R_1$ and $R_2$ are two radii of curvature; the pressure $P_j$ is bigger than the pressure $P_i$
1.1.3 Surfactants

To measure film properties, it is necessary to have a film which is stable in time. To reach it we add surfactants to a solution used for experiments. Surfactants are amphiphilic molecules, meaning they contain both hydrophobic groups (their tails) and hydrophilic groups (their heads) (see Figure 1.2). Once in solution, surfactants adsorb to the gas/liquid interfaces, thus they decrease the interfacial surface tension (from $\gamma = 72 \text{ mN.m}^{-1}$ to typical values $30-40 \text{ mN.m}^{-1}$) and they give it dynamic properties like interfacial viscoelasticity.

The surface tension is lowered with addition of surfactant until a concentration, the critical micelle concentration (CMC), is reached at which the surface is saturated with amphiphilic molecules. Above this concentration, the added surfactant molecules form aggregates, micelles, in the bulk and the surface tension becomes almost constant.

![Figure 1.2: Structure of a surfactant molecule](image)

Depending on the surfactant added to a solution, the gas/liquid interface will not have the same properties. There are two types of surfactants: soluble and insoluble. The first ones adsorb very quickly to interfaces and they can freely switch between an interface and the bulk. This characteristic of soluble surfactants makes the gas/liquid interfaces mobile [21], [13].

The second ones are solubilized in the micelles of a co-surfactant and they adsorb more slowly at an interface. During deformation the number of surfactants at the interface remains constant, thus making the interfaces much more rigid, changing the mobility condition to an incompressible condition.
1.1.4 Geometrical description

Plenty of geometrical properties of soap films and foams were studied by Plateau [43]. The most useful ones for this thesis are provided below.

Plateau’s laws

A foam consists of numerous bubbles packed together; the bubbles are put in contact which leads to a modification of their structure. To reach local equilibrium, we need to build the surface with minimal energy. At low liquid fraction, a foam is composed of polyhedral gas bubbles with liquid films residing between bubbles (Figure 1.3).

![Figure 1.3: 2D scheme of several polyhedral bubbles with liquid film separating them](image)

The films are smooth and they have constant mean curvature, which is determined by the Young-Laplace law. This is the first Plateau law [13].

Considering three connected bubbles, we see that the system reduces its energy by sharing part of the interface. So, the second Plateau law concerns how three films meet along an edge; they always meet forming angles of $120^\circ$. The edge at which three films meet is called a Plateau border.

Adding a fourth bubble to the system we have four edges connected at a vertex with tetrahedral symmetry. The angle of $109.47^\circ$ between the edges is the only possible choice that allows angles of $120^\circ$ between the films. This is the third Plateau law requiring equilibrium at vertices.
1.1. FILM STRUCTURE

Plateau border

One of the most important elements in foam formation is a Plateau border. This is the place at which three films meet, it means the channel available for a fluid flow which is bounded by films. A Plateau border has a triangular form with curvatures alongside (see Figure 1.4).

Figure 1.4: A Plateau border [13]; $r_1$ and $r_2$ are the radii of curvature of the liquid/gas interfaces; $r_{PB}$ is the length of one side of cross-section of the Plateau border

The shape of Plateau borders is determined by Young-Laplace law. If we consider $r_1$ - the radius of curvature in the plane perpendicular to the axis of the Plateau border (see Figure 1.4), $r_2$ - the radius of curvature in the plane parallel to the axis of the Plateau border, $p_1$ - the pressure of the liquid at equilibrium and $p_2$ - the gas pressure, we can write the Young-Laplace law:

$$p_2 - p_1 = \gamma \left( \frac{1}{r_1} + \frac{1}{r_2} \right). \quad (1.4)$$

We see on Figure 1.4 that $r_1$ is of the order of the Plateau border size $r_{PB}$ and $r_2$ is of the order of the radii of curvature the soap films. The difference $p_2 - p_1$ is the capillary pressure $p_c$ and for all examples considered in the following $r_1 \ll |r_2|$, so denoting $r_1=r$, we have from 1.4:

$$p_c \simeq \frac{\gamma}{r}. \quad (1.5)$$
Geometrical calculations let us determine the area of the Plateau border cross-section:

\[ S = \left( \sqrt{3} - \frac{\pi}{2} \right) r^2. \]  \hspace{1cm} (1.6)

When the radius of curvature \( r \) increases, the volume of liquid in the Plateau border increases also.

In the following we use two different notations for a Plateau border size. Both of them are schematically presented on Figure 1.5, where \( W_1 \) is used for Chapter 4 and \( W_2 \) for Chapter 5. They are linked to \( r_1 \) as:

\[ W_2 = r_1; \]  \hspace{1cm} (1.7)
\[ W_1 = \frac{\sqrt{3}}{2} r_1. \]  \hspace{1cm} (1.8)

For simplicity in both chapters we call the Plateau border size as \( W \).

![Figure 1.5: Notation for the Plateau border size; \( W_1 \) is used in Chapter 4; \( W_2 \) is used in Chapter 5](image)

**1.1.5 Mechanical properties of the interfaces**

In the previous section, we defined the liquid/gas interface and we gave notation for some geometrical properties. Applying a deformation to the surface we notice that surface tension does not keep its equilibrium value. In this section, we would like to explain surface tension variation as a function of applied deformation.

**Interfacial viscoelasticity**

Interfacial viscoelasticity is a response of an interface to shear and extensional stresses. There are two types of stress on an interface: expansion (dilatation or compression) and shear (see Figure 1.6). The viscoelastic properties of an interface change in the presence
of amphiphilic molecules; that is why it is important to take into account any exchange between bulk and surface. The interfacial viscoelasticity depends on the concentration and the solubility of the surfactant in water as well as on the ratio between mechanical excitation frequency and a characteristic time of diffusion and adsorption.

Figure 1.6: Two types of stress on the interface of the area $A_0$: expansion (dilatation here) and shear of the interface [41]

**Response to an expansion or compression**

If we expand a purely viscous interface of area $A_0$ at a rate $A_0^{-1} \frac{dA_0}{dt}$, it leads to surface stresses [13]:

$$\sigma_{\text{exp}} = \gamma + \eta_d \frac{1}{A_0} \frac{dA_0}{dt},$$

(1.9)

where $\eta_d$ is the surface dilatational viscosity (in kg.s$^{-1}$).

Additionally to a viscous response, the surfactants generate a surface dilatational elasticity. As illustrated in Figure 1.6, by stretching the interface $A_0$ by an amount $\delta A_0$, the surface concentration of the surfactants $\Gamma$ is reduced, which leads to an increase in the local surface tension value.

In the case of insoluble surfactants there is no exchange between bulk and interface.
During the surface expansion, the variation in concentration induced by the deformation is:

$$\frac{\delta \Gamma}{\Gamma} = -\frac{\delta A_0}{A_0}. \quad (1.10)$$

So, the interfacial response is purely elastic and it is thus linked to the Gibbs-Marangoni elastic modulus [29]:

$$E_{GM} = -\frac{d\gamma}{d\ln \Gamma}. \quad (1.11)$$

When the amphiphilic molecules are soluble, the exchange between bulk and surface depends on time-scale, which depends on the mechanism controlling adsorption dynamics. The surface elastic modulus becomes complex and depends on the excitation frequency $\omega$:

$$E_s^*(\omega) = E'_s(\omega) + iE''_s(\omega), \quad (1.12)$$

where $E'_s$ - the elastic modulus and $E''_s$ - the loss modulus linked to the surface viscosity by $E''_s = \omega \eta_d$.

If the oscillation frequency is very low with regard to a characteristic time of diffusion or adsorption, there is sufficient time for surfactant transfer to occur and for the surface concentration to adjust [13]. The surface concentration and the surface tension does not vary, so $E'_s(\omega) = E''_s(\omega) = 0$. In the limit of high $\omega$, surfactants do not have time to reach the interface during deformation, so $E'_s(\omega) = E_{GM}$ and $E''_s(\omega)$ vanishes.

In the intermediate case the model of Lucassen and Van der Tempel works well [32]. It assumes that surfactant exchange between the surface and the bulk is controlled by diffusion. So

$$E'_s = E_{GM} \frac{1 + \Omega}{1 + 2\Omega + 2\Omega^2}, \quad (1.13)$$

$$E''_s = E_{GM} \frac{\Omega}{1 + 2\Omega + 2\Omega^2}, \quad (1.14)$$

where

$$\Omega = \sqrt{\frac{D d c}{2\omega d\Gamma}} \quad (1.15)$$

with $D$ - diffusion coefficient in the bulk; $\Gamma$ - surface concentration; $c$ - bulk concentration.
1.1. FILM STRUCTURE

Response to shear

Analogically to an expansion, the surface shear stress can be defined. As presented in Figure 1.6 the strain is $\varepsilon_{\text{shear}} = \frac{\delta x}{L}$. The surface shear stress is linked to the strain by

$$\sigma_{\text{shear}} = \eta_s \dot{\varepsilon}_{\text{shear}},$$  \hspace{1cm} (1.16)

where $\eta_s$ is the surface shear viscosity and it depends on surfactants.

Like in the case of dilatation, the interfacial response depends on the excitation frequency and the complex modulus can be defined [13]:

$$G_s^*(\omega) = G_s'(\omega) + iG_s''(\omega),$$  \hspace{1cm} (1.17)

where $G_s'$ is the in-phase response to deformation, and $G_s'' = \omega \eta_s$ - the out-of-phase stress.

By the way, comparing to the previous case, there is no change in the interface area, so the surfactant concentration is not changed by the deformation. Only molecular interactions are responsible for elastic and viscous shear properties of the interface.

For soluble surfactants the shear elastic modulus is zero and the interfacial shear viscosity is small compared to the surface dilatational viscoelasticity.

Marangoni Effect

As was shown in [39] for experiments with increasing expansion rates, it is possible to find the rate at which the surfactants become less efficient at maintaining the low surface tension value. Indeed, it is necessary for the surfactants to be able to adsorb rapidly at the surface to form a dense layer there [13]. It influences the interfacial viscoelastic properties. Elasticity, in turn, acts to restore the interface. If an expansion is not uniform and if the surfactants do not have time to adsorb on the interface, then the interfacial concentration of molecules is not constant at the surface and it creates surface tension gradients. This produces a force in the plane of the interface ($xy$ plane) which counteracts the deformation by bringing back surfactants, called the Marangoni force [37]:

$$F = \frac{\partial \gamma}{\partial x}.$$  \hspace{1cm} (1.18)

This variation of surface tension produces liquid flow below the interface. This is the Marangoni effect.

To establish quantitatively these effects, we consider a piece of the interface $Ldx$ as
Figure 1.7: On the left: the Marangoni force [2], which counteracts the fluctuations by bringing back surfactants; on the right: the small piece of the interface $Ldx$, which is circled in red on the left; $F_1$ and $F_2$ - the surface tension forces; $F_v$ - the viscous force on the surface depicted in Figure 1.7, the viscous force acting on the surface from below is equal to

$$F_v = -\eta \frac{\partial U_x}{\partial z} \cdot Ldx \bar{e}_x,$$

where $U_x$ is a horizontal velocity field.

The surface tension force acts on both edges of the interface. On the right edge it is

$$F_1 = \gamma(x + dx)L\bar{e}_x$$  \hspace{1cm} (1.20)

and on the left edge

$$F_2 = -\gamma(x)L\bar{e}_x.$$  \hspace{1cm} (1.21)

Writing a force balance with respect to the $x$-direction we get:

$$\gamma(x + dx) - \gamma(x) = \eta \frac{\partial U_x}{\partial z} dx,$$

leading to the balance between the Marangoni and the viscous stress at the interface called the Marangoni law:

$$\frac{\partial \gamma}{\partial x} = \eta \frac{\partial U_x}{\partial z}.$$  \hspace{1cm} (1.23)

This equation shows that spatial variations of the surface tension generate a tangential stress at the gas/liquid interfaces and a velocity field in the subphase.
1.2 Film thickness measurement

In this section I present optical information about colors in a soap film which provides an important information about film thickness.

In optics, a thin film is a layer of material with thickness in a certain range (hundreds of nanometers to several microns in the context of this manuscript). When light rays reach the surface of a film, it is either transmitted or reflected at the upper surface [31]. Light that is transmitted reaches the bottom surface and may once again be transmitted or reflected. The light reflected from the upper and lower surfaces will interfere. The degree of constructive or destructive interference between the two light waves depends on the difference in their phase. This difference in turn depends on the thickness of the film, the refractive index of the film, and the angle of incidence of the original wave on the film.

![Figure 1.8: Scheme of light reflection on the thin film. $R_0$ is incident light ray from the light source $L_1$; $R_1$ and $R_2$ are reflected rays.](image)

The optical path difference of the reflected light can be determined as $CD = \delta$:

$$\delta = [SS_2] - [SS_1],$$  \hspace{1cm} (1.24)

where $[SS_2]$ and $[SS_1]$ are the optical paths defined on the scheme in Figure 1.8. Based on this $\delta$ we can find an expression for film thickness taking into account the normal angle of incidence of light waves:

$$\delta = 2nh \cos \theta_2,$$  \hspace{1cm} (1.25)

where $n = 1.33$ is the refractive index of water.

Interference will be constructive if the optical path difference is equal to an integer multiple
of the wavelength of light $\lambda$:

$$2nh \cos \theta_2 = m\lambda. \quad (1.26)$$

In our case the incident light is white light and interference patterns appear as colorful bands. Different wavelengths of light create constructive interference for different film thicknesses. Different regions of the film appear in different colors depending on the local film thickness.

Variation of film colors as a function of film thickness is represented on the color map assuming the normal angle of incidence of light waves (Figure 1.9). Due to gravitational effects and interference we can observe how a soap film becomes thinner through the changes of its colors.

![The colors of a soap film](image)

Figure 1.9: The color map of a thin soap film [1]

### 1.3 Soap film creation. Frankel’s model

In this section the process of isolated liquid film creation is considered.

When we withdraw a solid plate at constant velocity from a reservoir full of liquid, some liquid is entrained. The first quantitative analysis of this process was done by Landau, Levich and Derjaguin [36], [24], and much later in certain range of velocities [18], [38]. Several years later Frankel, Mysels and Shinoda [40] described, with a similar model, the half-thickness $h_{FR}$ of a stationary free soap film created by pulling a vertical frame out of a liquid bath. To solve the problem, several assumptions were made:
1.3. SOAP FILM CREATION. FRANKEL’S MODEL

1. inertial effects are neglected

2. the system is considered to be quasi-stationary;

3. the film surface is inextensible, so the velocity of liquid at the surface is supposed to be equal to the velocity of pulling;

4. gravity is negligible;

5. small variation of thickness in the dynamic meniscus allows us to use the lubrication approximation.

The problem is solved for half of the film (see Figure 1.10), where we can write the Stokes equation as:

\[
\partial_x p = \eta \frac{\partial^2 v_x}{\partial z^2}
\]

with \(\eta\) - viscosity of the solution and \(p\) - pressure.

From the notation on Figure 1.10 we can write for the pressure:

\[
p(x) = p_0 - \gamma \frac{d^2 h}{dx^2}
\]

where \(\frac{d^2 h}{dx^2}\) is the curvature of the interface. Taking into account the boundary conditions on the surface \(v_x = U\) at \(z = h\) and symmetrical boundary conditions \(\frac{dv_x}{dz}(z = 0) = 0\), we integrate the expression 1.28 to get the velocity profile:

\[
v_x(z) = \frac{\gamma}{2\eta} \frac{d^3 h}{dx^3}(x)(h^2(x) - z^2) + U.
\]

Now we consider the flux \(Q\) in the dynamic meniscus:

\[
Q = \int_{-h}^{h} v_x dz,
\]

then using the fact that the flux in the flat film is \(Q_{flat} = 2U h_{FR}\) and the steady-state assumption (imposing flux conservation \(\frac{dQ}{dx} = 0\)), the equation for the dynamical meniscus becomes:

\[
\frac{2\gamma}{3\eta} \frac{d^3 h}{dx^3}(x) h^3(x) = 2U(h_{FR} - h(x)).
\]
Next, we introduce dimensionless numbers $H = h/h_{FR}$ and $X = xCa^{1/3}/h_{FR}$, where $Ca = \frac{\eta U}{\gamma}$ - capillary number, so we have:

$$\frac{d^3H}{dX^3} = 3\frac{1 - H}{H^3}.$$  \hspace{1cm} (1.32)

The boundary conditions impose that film thickness at infinity is constant ($h_{FR}$) and the curvature of the flat part of the film is zero:

$$H(X \to +\infty) = 1$$  \hspace{1cm} (1.33)

$$\frac{dH}{dX}(X \to +\infty) = 0$$  \hspace{1cm} (1.34)

$$\frac{d^2H}{dX^2}(X \to +\infty) = 0.$$  \hspace{1cm} (1.35)
The curvature of the dynamic meniscus is equal to the curvature of the static meniscus in terms of large thicknesses $h \rightarrow +\infty$ at $X \rightarrow -\infty$:

$$\frac{1}{r_m} = \left( \frac{d^2 h}{dx^2} \right)^{\text{dynamic}},$$

(1.36)

where $r_m$ is the curvature of the static meniscus. So we can finally write Frankel’s law for the created film thickness:

$$2h_{FR} = 2.68 r_m Ca^{2/3}.\hspace{1cm} (1.37)$$

Numerous experimental and theoretical studies have focused on soap film entrainment by a solid frame. Frankel’s law is obeyed for octanoic acid and decanoic acid solutions (insoluble in water) in [35]; the bulk liquid in the film is considered as the second surfactant reservoir and surfactant transport is coupled to exchanges with micelles due to bulk diffusion in [17], [27]; the modification of the Frankel analysis to include an interfacial slip condition in [5] successfully captures the range in exponents and fitting parameters obtained experimentally; the film thickness follows Frankel’s law only for the lower part of entrained film in [47], [45] and for large capillary numbers the thickness is smaller than predicted by Frankel because the surface becomes mobile. At the high velocities of entrainment, inertial effects should be taken into account [19].

The main assumption made by Frankel is the incompressibility of the interfaces: the velocity of the interface is uniform and is equal to that of the solid frame. In order to satisfy this condition, interfacial stresses were computed \textit{a posteriori} in Frankel’s framework, once the velocity field in the film had been determined [12]. These stresses arise from the stretching of the surfactant-covered interfaces. The effect of finite interfacial stretches on the thickness of the extracted soap film is described in [49] by considering viscoelastic properties of the interface which can explain the discrepancies between Frankel’s law and experiments. To stay rigid the interface has to be able to support the interfacial stress gradient $\delta \sigma$, which is possible in the case of large elasticities or interfacial viscosities. In that case, the interfacial stress difference between the steady thin film and the meniscus is equal to [49]:

$$\delta \sigma = 1.85 \gamma (3Ca)^{2/3}.\hspace{1cm} (1.38)$$

For capillary numbers in the range $10^{-6}$ to $10^{-3}$, $\delta \sigma$ corresponds to surface tension gradients between $10^{-2}$ and $1\text{mN}\cdot\text{m}^{-1}$. This result is valid only for incompressible interfaces.
Elastic interfaces lead to a steady solution with a film thickness smaller or equal to that predicted by Frankel which is in agreement with experimental results obtained by [15], [35], [47], in contrast, a solution with purely viscous interfaces cannot form a steady film [49].

Combining experimental works for surface tension gradients in the upper part of a film pulled vertically [46] and in the lower part [14], it is possible to predict the surface tension difference across the whole film. Recent papers on Frankel’s experiment [56], [4] predict a very small value of surface tension gradient along the film.

Another interesting case is an extension of a suspended horizontal soap film. Firstly, the film extends homogeneously, then its extension stops, and a new thicker film is extracted from the meniscus [50]. This complex behavior is explained using a model based on Frankel’s theory and the interface rigidification resulting from confinement.

1.4 Deformations

One of the objectives of this thesis is to observe processes happening between connected foam films under deformations. Why is this question interesting? It is known that liquid foams are very dissipative materials, their effective viscosity is very high. This fact leads to their wide use and production in the industry: in food, in cosmetics, as drilling fluid in the oil industry [52] etc. It was shown that structural disorder and film-level flow linked to interfacial rheology are the possible origins of high energy dissipation but the question about these local processes is still open [16]. Between the two surfaces of the thin film and in menisci the viscous liquid phase is confined and it has a strong influence on the dissipation rate. And the dissipation rate also depends on the interfacial properties. Therefore, local flows have a dependence on the interfacial stresses and the local film thickness and to measure them in a 3D sheared foam is impossible.

That is why it is needed to predict the dissipation rate by considering different models at the bubble scale which can lead to predictions at the foam scale. The interplay of capillary effects and viscous friction in the foam films formed between neighbouring bubbles is considered in [20], [54], [22]. Flow models in [33] and [34] assume that all of the continuous phase is contained in the films and that there is no exchange of fluid between the films so the film surfaces are mobile. These models predict a very small contribution of viscous dissipation in the films, unless the shear rate is exceedingly large, which is not the case in the experiments, for example, for oil-in-water emulsions [44]. Another model taking into account that the contribution of viscous dissipation, relative to the elastic work done in deforming a foam, can
be quite significant is presented in [48] and complemented in [11] by viscosities calculation.

The question of the film thickness measurements or deformations at the bubble scale is still open to distinguish different assumptions presented in the models on the local flow. Considering T1 process (four objects swap neighbours, see Figure 1.11), the effect of the viscoelastic parameters on the dynamics was experimentally and theoretically investigated in [26]. It was shown that the relaxation after a T1 is mainly driven by interfacial viscoelasticity, but not by the shear viscosity of the bulk liquid. This work was recently extended by [57] to allow film curvature and variations in surface tension on all films.

Figure 1.11: A T1 process [3]. Objects A and B are in contact and objects C and D are on either side of the AB group and touching both A and B. The T1 process consists in breaking the contact between A and B and establishing the contact between C and D.

Some experiments involving the measurement of the thickness of the dynamic wetting films formed between bubbles and moving solid wall were made in [23].

Other experiments showing the deformations at the bubble scale were made by [6]. Two hemispherical bubbles in contact were investigated experimentally in static and dynamic adhesion. The dynamic contact angle was found to differ from a static one, which indicates a variation of surface tension. These measurements may be used to predict the contribution of the films and vertices to microscopic foam rheology. The model to predict the fast linear viscoelastic response of bulk foam based on these experiments is discussed in [7].

A more complicated system consists of a bubble cluster was developed in [8]. The authors formed a bubble cluster between two plates and induced a T1 by a translation of the bottom plate. The rearrangement duration is a key parameter to describe how the microstructure dynamics control the macroscopic rheological response. This duration has been measured on the cluster and shown to be proportional to a ratio between the dilational surface viscosity and the surface tension. The stability aspect in the T1 event is also considered in [9] because it is crucial for foam stability understanding. More recent experiments on the T1 events are discussed in [42] by measuring the film thickness using light adsorption and by measuring velocities in a stretched film by particle tracking. Both of these parameters depend on two different behaviors of the film (homogeneous extension or resistance to elongation with a new film created from the meniscus) but the strong characteristics to distinguish these two cases...
are not found.

The question about the influence of neighbours to the film during deformation or the amount of interface area which is created/destroyed in the meniscus is still open. Recently it was considered by [10] to provide the tangential velocity induced in a film by the deformation of its first neighbour.
Chapter 2

Three films set up and optical tools

To deform liquid foam on a sample of several connected liquid films it is required to create a set up which allows us to stretch or to compress a chosen liquid film. Also, it must be possible to control the velocity of these deformations and the amplitude of pulling or pushing.

The construction of a deformable frame has to be done in such a way to have the possibility to install recording devices like cameras. The method of specular light reflection with a light source is applied. Thereby, there is a need to have access to thin film for several equipment at the same time to measure physical parameters such as film thickness or Plateau border size.

In several experiments control of Plateau border size was done with an infusion of chemical solution directly in the meniscus using a syringe pump for this action.

The idea of the following experiments intends to control gravity. For this purpose, it was necessary to separate the experiments into two parts: on a rigid plane table and a motorized inclined table. Nevertheless, the basic deformable frame and mechanism of deformation are the same.

2.1 Creating and deforming foam films

In this section, I explain all the frame configurations, both mobile and immobile, used for the three films experimental setup, the methods to control the frame motion, the Plateau border size and the angle of inclination of all the systems.
CHAPTER 2. THREE FILMS SET UP AND OPTICAL TOOLS

2.1.1 Frames design

The main idea of the first two setups is to develop a frame which lets us create three soap films with one common meniscus. Creation of the films requires to combine both immobile and mobile frames. Schematic representations of two different versions of the set up are shown in Figure 2.1 and Figure 2.2 (denoted in the following as SET1 and SET2). First of all, it was needed to make the three films at 120°, for this purpose we use immobile frames marked by red lines on both schemes. We create the three flat films $F_1$, $F_2$ and $F_3$ which are connected through the common meniscus and which are obeying the Plateau laws. We want to vary the size of the film $F_1$ from $L_0$ to $L_0 + \Delta L$. In this aim, we use a mobile part which is designed to modify the size of the film $F_1$. On the schemes, it corresponds to the piece $P_3$. Deformation is done at a constant velocity and is controlled by a linear stage $M$.

Design of plastic pieces was made with the free software OpenSCAD. The immobile frame (red on Figures 2.1 and 2.2) consists of two parts $P_1$ and $P_2$. $P_1$ is made of two parallel immobile pieces illustrated in Figure 2.3a. The two films $F_2$ and $F_3$ are not deformable. They are limited by the two pieces $P_1$, a rigid block $P_2$ (see Figure 2.3b) connected to the $P_1$ pieces, and a free meniscus.

![Figure 2.1: Schematic representation of the SET1 on a reclining table ($\theta \in [0^\circ; 9^\circ]$); $F_1$ - deformable film, $F_2$ and $F_3$ - undeformable films; $P_1$ and $P_2$ - immobile frames, $P_3$ - mobile frame controlled by linear stage $M$; $L_0$ - initial length of $F_1$; $w$ - its width](image)
2.1. CREATING AND DEFORMING FOAM FILMS

Figure 2.2: Schematic representation of the SET2 on a horizontal table; \( F_1 \) - deformable film, \( F_2 \) and \( F_3 \) - undeformable films; \( P_1 \) and \( P_2 \) - immobile frames, \( P_3 \) - mobile frame controlled by linear stage \( M \); \( L_0 \) - initial length of \( F_1 \); \( w \) - its width

The deformable film \( F_1 \) is limited by the free meniscus, the two pieces \( P_1 \) and a mobile piece \( P_3 \). This piece has to be designed in a way that we can immerse the system into solution as shown on Figures 2.4a and 2.4b for SET1 and for SET2 respectively. The piece \( P_3 \) for vertical case lays in the same plane as a deformable film \( F_1 \) and it was designed the way presented on Figure 2.6a. For the horizontal situation (Figure 2.1) \( P_3 \) lies in the \( xy \)-plane perpendicular to the plane of \( F_1 \). In order to deform the film \( F_1 \), the piezo stage \( M \) has to be in the same plane as the film (\( xz \)-plane), that is why the piece \( P_3 \) for the SET1 differs a bit from the one for the SET2 (see position of \( M \) in Figures 2.1 and 2.2).

In addition, the printed mobile frame \( P_3 \) has a width of 2 mm otherwise it is not rigid enough. The film \( F_1 \) has to lay in the plane \( xz \) but this frame thickness leads to an uncertainty on the meniscus position. On Figure 2.5 (view of the set up from the side) we present a solution to this problem by attaching a long triangular prism (designed piece for printing is on Figure 2.6b), it allows us to fix the meniscus of \( F_1 \) and to keep \( F_1 \) in the \( xz \)-plane.
2.1.2 Printed and installed set up

For producing of all these pieces we used 3D printer *Ultimaker*² and PLA (poly lactic acid) plastic. It is easy and fast to reprint pieces if they are broken. There is no risk of corrosion of plastic pieces during long contact with the chemical solution. Finally, fast printing and replacing can allow us to use several types of solution without any risk of substance ingress of one solution to another.

Immersing all the system into the reservoir filled with the chemical solution requires us to design and to print a reservoir too. We used two different types of reservoir material: PLA and glass (made with the help of glass department of IPR). Both of them have a down side. The plastic one leaked a bit during experimental session while the glass one is fragile and it is not resistant to the cleaning process (dissolution of glue between glass pieces during cleaning with acetone and ethanol when changing chemical solution composition).
2.1. CREATING AND DEFORMING FOAM FILMS

Figure 2.5: Side view in the $yz$-plane of the three films; left: the meniscus is attached on a long triangular prism; right: a possible situation without triangular prism

(a) Mobile frame $P_3$ for the vertical set up

(b) Long pyramid for meniscus fixation on the frame $P_3$

Figure 2.6: Plastic pieces for soap film creation

The full set-up SET2 is shown in Figure 2.7a, and the process of immersing it into the reservoir is presented on Figure 2.7b. Some plastic pieces are screwed to each other, some of them are glued together for all set ups.

2.1.3 Motors and controllers

The processes of stretching and compressing a soap film are controlled by the amplitude and the velocity of the mobile frame motion. To control these parameters we use a compact linear stage with ultrasonic piezo motor PI using PIMicroMove software. The minimum incremental motion given by the piezo stage is $0.3 \, \mu m$ and the maximal velocity is $250 \, mm/sec$. To verify the data given by a software, we perform a series of experimental tests
CHAPTER 2. THREE FILMS SET UP AND OPTICAL TOOLS

(a) Photo of SET2 with soap films created on it

(b) Soap film creation with PLA reservoir

Figure 2.7: Soap film creation

for the piezo motor velocity (denoted by \( U \) in the following). Using a high speed camera we record the motor motion at constant velocity \( U \) displaced at distance \( \Delta L = 13 \text{ mm} \). For velocities \( U \) smaller than 100 mm/sec, the values given by the software corresponds well with the experimental ones. For the values bigger than 100 mm/sec we noticed that the actual motor velocity is smaller than the one given by the software. So, we calibrated \( U \) for high values. The calibration for velocity \( U = 250 \text{ mm/sec} \) (given by the software) is presented on Figure 2.8. Also, on Figure 2.8 we see that the velocity \( U \) is constant and no acceleration period is visible. Hence, we can provide actual piezo motor velocity in the Table 2.1 and in the following we call motor velocity \( U \) the actual value obtained using calibration.

Figure 2.8: Calibration graph for theoretical motor velocity \( U = 250 \text{ mm/sec} \)
2.1. CREATING AND DEFORMING FOAM FILMS

<table>
<thead>
<tr>
<th>$U_{\text{theor}}, \text{mm/sec}$</th>
<th>$U_{\text{exp}}, \text{mm/sec}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>88</td>
</tr>
<tr>
<td>150</td>
<td>102</td>
</tr>
<tr>
<td>200</td>
<td>112</td>
</tr>
<tr>
<td>250</td>
<td>112</td>
</tr>
</tbody>
</table>

Table 2.1: The values of piezo motor velocity $U$ given by software ($U_{\text{theor}}$) and measured during tests ($U_{\text{exp}}$)

2.1.4 Syringe driver

Several experiments require us to keep a stable size of Plateau border or a stable soap film thickness in time. For this purpose, we used a syringe driver or syringe pump which is a small infusion pump to gradually administer small amounts of fluid directly to a meniscus. We use Remote Infusion Only PHD ULTRA™ Syringe Pumps from Harvard Apparatus. Four syringes are installed and we tested a pump at a range of 0.001 ml/min to 1 ml/min for each syringe. For our experiments, we used values of around 0.05 ml/min. The infusion was made directly into the meniscus located on the top edge of the $P_1$ pieces by four needles connected to four syringes $S_{g1}$, $S_{g2}$, $S_{g3}$ and $S_{g4}$ as pictured on Figure 2.9.

![Figure 2.9: Scheme of chemical solution infusion into the meniscus of films $F_2$ and $F_3$ through syringes $S_{g1}$, $S_{g2}$, $S_{g3}$ and $S_{g4}$](image-url)
2.1.5 Motorized table

In order to study a gravitational instability, we needed to vary the gravity projection in the $F_1$ film plane. Its value is $\bar{g} = g \sin \theta$, with $\theta$ the angle shown in Figure 2.1. At the same time the incidence angle for the spectral camera and the light reflection method for the color camera have to work as well. The best solution is to change the angle together for the whole system. This is possible by using a motorized table. At the initial position (zero degree) the table is horizontal. Using the motor, which turns the plane of the table relative to its axis of symmetry laying on the middle of the plane of the table, we can modify the inclination up to the biggest possible angle for our configuration; and it is $9^\circ$. Indeed, with the current set up, it is impossible to create a thin film at higher angles because of the geometry limitation presented on Figure 2.10 right.

![Figure 2.10: Geometry of the system SET1 at the angles 0 and bigger than 9 degrees](image)

Thereby, in the plane of the film we keep the same angle of inclination for both color and spectral cameras turning all the system all together.

Other experiments were carried out using a flat laboratory table. In this way, we modify the angle of inclination $\theta$ from 0 to $9^\circ$ for SET1 (Figure 2.1) and we have $\theta = 90^\circ$ for SET2 (see Figure 2.2).
2.2 Chemistry

In these studies we used several chemical surfactants, both anionic and cationic. All of them were ordered from Sigma-Aldrich. To make an interface of a film more rigid we add an insoluble-in-water molecule - dodecanol ($C_{12}H_{26}O$). Let’s consider the first - cationic surfactant: Trimethyltetradecylammonium bromide (TTAB) with a molecular weight 336.39 g/mol. The critical micelle concentration in pure water is equal to $3.6 \cdot 10^{-3}$ mol/L (1.5 g/L [28]). The first compound consists of TTAB 10 g/L (6.7 CMC) with 30 % of Glycerol (Figure 2.11). We used this solution both with addition of dodecanol (0.05 g/L) and without.

![Chemical formula of the first compound: Trimethyltetradecylammonium bromide (TTAB) and Glycerol](Figure 2.11)

The second anionic surfactant we used is Sodium dodecyl sulfate with molecular weight 288.38 g/mol; the critical micelle concentration in pure water is equal to $8 \cdot 10^{-3}$ mol/L. This compound consists of SDS 5.6 g/L (twice the CMC), glycerol - 15 % of volume (Figure 2.12). As previously, we made solutions both with dodecanol (0.05 g/L) and without.

All the surfactants were prepared with the same protocol. Powder of SDS or TTAB was weighed on the balance, then it was placed into reservoir. After this distilled water and glycerol were added and all the solution was placed on the magnetic agitation for several hours to mix and to dissolve surfactants. If we made a compound with dodecanol, it was weighed the same way and added at the stage together with SDS or TTAB.

![Chemical formula of the second compound of Sodium dodecyl sulfate](Figure 2.12)
To conclude, I provide all the chemical solutions used in the following experiments in Table 2.2. Each chemical solution is denoted by a number which is used in the following. Values of \( \gamma \) were measured using the pendant drop method by myself, data for \( \mu_s \) and \( \eta \) are taken from [25] for \( S_3 \) and \( S_4 \) and from the agreement theoretical and experimental data in Chapter 4 for \( S_1 \) and \( S_2 \) solutions.

<table>
<thead>
<tr>
<th>Property</th>
<th>TTAB (( S_1 ))</th>
<th>TTAB+dod (( S_2 ))</th>
<th>SDS (( S_3 ))</th>
<th>SDS+dod (( S_4 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma ), ( 10^{-3} ) N m(^{-1} )</td>
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<td>29</td>
<td>29</td>
<td>30</td>
</tr>
<tr>
<td>( \rho ), kg m(^{-3} )</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>( \eta ), ( 10^{-3} ) Pa s</td>
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<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>( \mu_s ), kg s(^{-1} )</td>
<td>( 0.8 \times 10^{-7} )</td>
<td>( 0.8 \times 10^{-7} )</td>
<td>( 0.6 \times 10^{-7} )</td>
<td>( 1.9 \times 10^{-7} )</td>
</tr>
</tbody>
</table>

Table 2.2: Chemical solutions and its properties used for the experiments. The notation: \( \gamma \) is the surface tension; \( \rho \) is the density; the bulk viscosity \( \eta \) and the surface shear viscosity \( \mu_s \)
2.3 Observing foam films. Optics

In this section, I would like to consider several optical devices used in the experiments. The goal is to have the possibility to control the Plateau border size, observing colors of an entire soap film during and after being stretched or compressed and to measure film thicknesses.

The experimental tools were improved significantly during research. At the beginning of my PhD, we started with a simple webcam, then with the video option of a photo camera and, finally, with an industrial color camera.

For soap film thickness measurements, we started with a spectrometer Flame of Ocean Optics which gives us a spectrum at a single point of the thin film surface. The main difficulty was the following: having spectral information in one point only, we cannot see the thickness evolution of the entire soap film in time. I performed several series of experiments using the spectrometer but then the technique was improved with a hyperspectral camera so I will not provide any results of experiments with a spectrometer because the data from hyperspectral camera are more reliable and more quantitative.

2.3.1 Long range microscope

As shown in Figure 2.7a three flat soap films are connected along a free meniscus. Understanding the influence of meniscus size on the effects which are taking place in a thin film under deformation is interesting for us. The geometry of the Plateau border is explained in the previous Paragraph 1.1.4. The measurements of the Plateau border size are done for different geometry: in Chapter 4 and Chapter 5. On Figure 1.4 we have $W_1$ for the meniscus size used for the geometry from Chapter 4 and $W_2$ for the one from Chapter 5.

According to the geometry of all the setups (millimetric size in $xz$ plane) observation of Plateau border size is quite tricky. We need to get the biggest possible magnification but the focal distance still has to be big enough to give access for other devices to be installed.

For the geometry used for experiments in Chapter 5 we use a 105 mm lens in normal position plus a special bellows (accordion-like, pleated expandable part of a camera). We get magnification of 5.25 times. In the plane perpendicular to the camera we set up the light. The typical image from the camera is shown in Figure 2.13b, with $W$ - the Plateau border size. The main disadvantage of this system is that it is massive and heavy.

A next step to improve the set up was to use a long-range microscope. We have chosen long-range microscope Navitar 1-50486 with 12x zoom together with a collimated light source.
CHAPTER 2. THREE FILMS SET UP AND OPTICAL TOOLS

The microscope is used for the experiments described in Chapter 4. An example of the image from the microscope is presented in Figure 2.13a. On the image 2.13a we see from a direction perpendicular to a film (see Figure 1.5 left) and in Figure 2.13b we see from a direction parallel to a film (see Figure 1.5 right).

![Figure 2.13a](image1.png) ![Figure 2.13b](image2.png)

Figure 2.13: The picture of the Plateau border obtained from (a) long-range microscope and (b) lens with special bellows; $W$ - Plateau border size

2.3.2 Color camera

The color camera is used to observe the entire film before, during and after being stretched or compressed. We record a film in the following way.

The flat soap film is illuminated by a square (100 mm $\times$ 100 mm) white LED lamp Phlox® with minimal luminance 50000 $cd/m^2$. At the reflection point we install the camera to get a colored image of the thin film $F_1$ as shown in Figure 2.15. The angle of incidence $\theta_1$ of light rays is the same as the angle of inclination of the camera, so the reflected light rays from the film $F_1$ go to the camera. The film $F_2$ is in the field of view too, but light rays pass this plane at another angle of incidence, that is why on the entire image we see $F_2$ as a black surface because no light is reflected to the camera (Figure 2.15).

Light waves reflected by the upper and lower interfaces of a thin film interfere with one another, either enhancing or reducing the reflected light. Thus, color camera images show multiple colors on the soap film which correspond to certain values of film thickness (Equation 2.1, page 38). One image example is shown in Figure 2.14. The free meniscus is located at $z_0$, the supported one at $z_m$. The colors can be correlated with film thickness using Figure 1.9.
2.3. OBSERVING FOAM FILMS. OPTICS

Figure 2.14: Different colors on thin film due to light interference

As a camera we use IMAGING SOURCE DFK 23UM021; frame rate for all the experiments is set at 80 fps. Each image provides us information about film thickness on the entire film before, during and after experiments. The maximal distance on which we pull the film (the amplitude) is 13 mm. Thus, at the fastest velocity of pulling (around 100 mm/sec) the frame rate of the camera lets us get around 10 frames during the pulling process. This amount is sufficient to extract some data like the front velocity of an extracted film (see Chapter 3).

For one series of experiments we use the Schneider c-mount lens - XENOPLAN 1.4/23-0902 with focal length 11 mm and maximum aperture f/1.4. For another - one with focal length 50 mm and maximum aperture f/1.4; the minimum focus distance is 0.45 m.

2.3.3 Hyper spectral camera. Description

The main difference from the experimental set up with color camera is the possibility to measure a soap film thickness not only qualitatively but also quantitatively. The hyperspec-
tral camera Resonon Pika L is used for this purpose. The camera creates a digital image containing spectral information. For each spatial point along a line $L$, the hyperspectral image provides a continuous curve of incoming light intensity versus wavelength. As shown in Figure 2.16 the chosen line $L$ of pixels is oriented along the direction $z$ in the plane of the thin film $F_1$. The choice of the positioning of the line $L$ is not very precise but the line has to be around the middle of the film $F_1$.

![Figure 2.16: Scheme of light reflection on the film $F_1$ for color camera and on the line $L$ for the hyperspectral camera](image)

The incident light from the incandescent lamp is partly reflected from the plane of a soap film $F_1$ and, to obtain hyperspectral data, the light signal from each point of the line is dispersed into its spectral components like passing the light from each point through a prism. This process occurs for every point in the line $L$. The dispersed light from all the points is imaged onto a focal plane array, all the spectral (color) information is collected at the same time for each pixel. The result is a detailed spectral curve for every spatial pixel in the image. The raw image of a soap film from the hyperspectral camera is presented in Figure 2.17. The horizontal direction is the spatial coordinate $z$, and the vertical one is the wavelength $\lambda$ of the light from 375 nm at the top to 1010 nm at the bottom. The grey level is the light intensity (in arbitrary unit). The spatial resolution of each point along the line $L$ is 53.8 \(\mu\)m using the objective XENPLAN 1.4/23-0902.

Each time moment we record 2D matrix of data containing spatial and spectral information (intensity), the maximal frame rate is 296 fps. Collecting all the data altogether we have 3D data cube containing as well temporal information.
2.3.4 Relationship between light intensity and film thickness

Let’s consider a case of a plane-parallel layer with the refractive index of water $n = 1.33$ and the geometrical thickness $h$ (Figure 1.8). Starting from the point light source $L_1$, a ray $R_0$ is partially reflected from the upper film surface at the point A (ray $R_1$ at angle $\theta_1$) and partially refracted into the layer (at angle $\theta_2$). At the lower boundary of the layer, the ray is reflected again from the bottom surface at the point B and refracted at the point C. Finally, the ray $R_2$ leaves the upper boundary layer parallel to $R_1$ and exits into the air again. Further reflections in the layer refract the ray $R_0$ infinitely and divide it into parallel rays with strongly decreasing intensity. Since all reflected and refracted rays have their origin in the $R_0$ ray, they are coherent and can thus interfere with each other. Depending on the path difference $\delta$, the two main reflected rays $R_1$ and $R_2$ may interfere with each other.

The light intensity is proportional to the optical path difference as $I(\lambda) \sim \cos\left(\frac{2\pi}{\lambda} \delta\right)$, where the optical path difference $\delta$ is following (see Figure 1.8):

$$\delta = n(AB + BC) - AD.$$ 

Considering light rays geometry we can find $AD$ from the triangle $ADC$ as $AD = AC \sin(\theta_1)$ and $AC$ from the triangle $ABC$ as $AC = 2h \cdot \tan(\theta_2)$. Later, we know that
\[ AB = BC = \frac{h}{\cos(\theta_2)}. \] So, finally, we have:

\[ \delta = 2h \cdot \sin(\theta_1) \cdot \tan(\theta_2) - \frac{2nh}{\cos(\theta_2)}. \]

Next, using Snell’s law, \( \sin(\theta_1) = n \cdot \sin(\theta_2) \), and trigonometric transformations we get:

\[ \delta = 2h \sqrt{n^2 - \sin^2(\theta_1)}, \]

which gives the final expression for the light intensity, which lets us compute the film thickness:

\[ I(\lambda) \propto 1 - \cos \left( \frac{4\pi hn}{\lambda} \left( 1 - \frac{\sin^2 \theta_1}{n^2} \right)^{1/2} \right). \] (2.1)

### 2.3.5 Hyperspectral camera. Positioning and synchronization

As it was discussed above, the hyperspectral camera collects data along one line on the thin film so here I explain how to find the position of this line because it is not obvious. According to the geometry of the experimental set up (SET1 and SET2), the soap film \( F_1 \) has an initial size 50 mm \( \times \) 19 mm (length \( \times \) width). Using these dimensions, the following piece of paper was printed and glued on a piece of carton (Figure 2.18).

We place it on the frame where soap film should be created, and we record data with the hyperspectral and color cameras. The resulting image is shown in Figure 2.19.

![Figure 2.18: The piece of paper used for finding spectral camera position on thin film, 50 mm \( \times \) 19 mm; the diagonal boundary between red and green satisfies \( z = -0.42x + 21 \); line \( L \) is at the position \( x = 40.14 \) mm](image)

The width of a paper piece is shown in the z-direction. Comparing the images 2.18 and 2.19 of our test-piece, the distance between \( z_0 \) and \( z_g \) agrees with a tiny horizontal green line on the top of the test-piece. Then, we see the part from \( z_g \) to \( z_r \), which is intensified at
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Figure 2.19: Resulting spectrum image at each wavelength for one line of spectral pixels on the plane of the piece of paper with red and green colors

around 640 nm which fits wavelength of red, next, green again (from \( z_r \) to \( z_{g1} \)) and tiny red (from \( z_{g1} \) to \( z_1 \)).

The boundary between both colored domains satisfies \( z_{\text{diag}} = -0.42 \ x + 21 \) (in mm). Measuring the distance between \( z_0 \) and \( z_1 \) on the spectral image (Figure 2.19), we get the scale of the hyperspectral camera. Then, measuring the distance between \( z_0 - z_r \) and \( z_r - z_1 \), we deduce the lateral position \( x \) of the observed line \( L \) with respect to the bottom left corner of the test-piece (using the equation of the boundary).

For this test image the line \( L \) is at the position \( x = 40.14 \) mm. So, now on the corresponding image of test-piece taken by color camera we know the position of line \( L \) at \( x = 40.14 \) mm and we can correlate it with the position of \( L \) on the soap film.

Both cameras (colored one and hyperspectral) are controlled (triggered) by Matlab and they are synchronized at time moment \( t_{\text{init}} \). The code allows us to save time moments for each frame both for the color camera and for the hyperspectral one. As the hyperspectral camera is faster, we need to find a way to make a correspondence between one frame of the color camera and a frame of spectral one taken at the same time moment \( t_i \). For the experiments besides the time from the beginning of the experiment, we save the time value
as hour-minute-second/etc for each frame both for the color and the hyperspectral cameras. During the analysis, it permits us to easily find the correspondence between frame numbers of both cameras.

To reduce the amount of saved data we use ROI (region of interest) positioning for both cameras and in this way we cut images to save only areas with visible parts of soap film.
Part II

Three films set-up
Chapter 3

Frankel’s film extraction

In this chapter, I would like to explain the processes happening in the liquid film under stretching. I carried out experiments to characterize qualitatively front velocity between the thick and the thin films during and after deformation. With my experiments, I’ve just started to answer this question and it was developed afterward by postdoc Adrien Bussonnière.

It was noticed that the front between the thick and thin films is unstable under gravity, and this question became the main one for my research. It is discussed in the next chapter called Rayleigh-Taylor like instability.

3.1 Introduction

Let us consider a rectangular film from the configuration (SET1) illustrated in Figure 2.1, which we want to deform by increasing the area of the film $F_1$ by pulling the mobile part of the frame $P_3$. For this series of experiments, we use the angle of inclination $\theta = 0^\circ$. The frame construction is shown in Figure 3.1. On the left image the films $F_1$ and $F_2$ are visible, on the right one - $F_1$ and $F_3$. We are interested in film behavior during and after deformation so we have to install the color camera $C_1$ (see Paragraph 2.3.2) for a recording of the entire soap film $F_1$ and the hyperspectral camera $C_2$ (see Paragraph 2.3.3) at the incidence angle $45^\circ$ to $xz$-plane for film thickness information (see Figure 3.2). The recorded line $L$ for the hyperspectral camera is drawn on the left image 3.1.

During this series of experiments, we do not control or measure the Plateau border size as working with the three cameras together was difficult for simple steric reasons.

The deformation of the film is done with the piezo stage M described in 2.1.3. It moves and deforms the film $F_1$ at constant velocity $U = 80$ mm/sec with an amplitude $\Delta L = 13$
CHAPTER 3. FRANKEL’S FILM EXTRACTION

Figure 3.1: Frame configuration in horizontal position SET1

Figure 3.2: Scheme of the SET1. $L_1$ - LED lamp Phlox®; $C_1$ - the color camera IMAGING SOURCE DFK 23UM021; $L_2$ - incandescent lamp; $C_2$ - spectral camera Resonon Pika L. Initial film size is $L_0$ and we stretch it to $L_0 + \Delta L$; the film of interest $F_1$ of the width $w$.

mm. Two different chemical solutions were used: $S_3$ and $S_4$ (see Table 2.2).
3.2 Experimental protocol

Most of the experimental protocol was automatized with Matlab.

1. First, we create a soap film using a reservoir filled with the chemical solution and wait 15 seconds for a thin film to stabilize at a thickness $h_0$ of quasi-uniform value in the range 0.5 - 1 $\mu$m. Initial film size $L_0 = 6$ mm.

2. Next, we run both cameras with Matlab at time moment $t_0 = 0$ sec.

3. Using the software for the piezo motor we run it at $t_0$ with velocity $U = 80$ mm/sec and amplitude $\Delta L = 13$ mm (the distance in the $z$-direction on which we displace mobile frame). The motor stops at $t_s$. The total film size becomes $L_0 + \Delta L = 19$ mm.

4. Wait 10 seconds to record the data after $t_s$ moment (if it is possible, sometimes a soap film breaks earlier than $t_s + 10$).

5. Finally, save all the data: 3D cube from hyperspectral camera (space, time and intensity for all wavelength) and 4D matrix from color camera (space $x$-direction, space $z$-direction, time and RGB value). Also, the time for each frame of both cameras.

6. We repeat the experiment three times for each chemical solution and repeat all series twice.

3.3 Qualitative observations

Immersing the frame into the chemical solution leads us to the creation of three rectangular flat soap films meeting at the meniscus. We focus on the film $F_1$ laying in the horizontal $xz$-plane. Let's consider the observations at successive times after its formation, as shown in Figure 3.3a.

On the image presented in Figure 3.3a we see two menisci: the free meniscus (the one between three connected flat films) at the point $z_0$ and the supported meniscus (the one on the mobile solid frame) at the point $z_m(t_0)$. The film has a width $L_0 = 6$ mm and length 50 mm. Film colors are bright but rather dispersed from 300 nm to 400 nm according to the color map Figure 1.9. On Figure 3.3b we see the corresponding thickness profile at the same time (the data from hyperspectral camera). As we can see, the data for film thickness values are in good correspondence with what we estimated by the color map. The closed system, which constituted $F_1$ initially, is called "the initial film" with a width $d_1$ and a thickness $h_1$. 
Figure 3.3c shows the behavior of the film during the motion. During the described deformation the total area of the film increases. The free meniscus is always at position $z_0$, the supported one is moving and we define its position as $z_m(t)$. The area of the film is getting larger, and we see the appearance of two horizontal symmetrical grey parts close to both menisci while the colors of the film in the middle are still bright. On Figure 3.3d for thickness profile we see that the grey parts correspond to high thickness (3-4 microns) while the part in the middle keeps the thickness values slightly smaller than on Figure 3.3b.

Indeed, the thin part of the film has a constant volume and does not exchange liquid with the extracted thicker film. When the surface area increases, the thickness decreases.

In order to get thicker films extracted at $z_0$ and $z_m(t)$, the chemical solution, which was in the meniscus before, is used. Based on the discussion in Chapter 1.3, we call the extracted film the Frankel’s film of width $d_2$ with the velocity of extraction $V_{fr}$. This film has the thickness $h_2$ and in a steady situation it is controlled by the velocity $V_{fr}$ and the meniscus size $r_m$ according to equation 1.37.

Next, on Figure 3.3e we have a stretched film and the piezo motor has already stopped, $t > t_s$. The extracted films are symmetrical and they have more or less the same width. After the motor stop, $d_2$ keeps growing from both menisci.

The boundary between the initial film and the Frankel’s film is straight and sharp. It is well defined during the piezo stage motion and afterwards. That is why we can characterize and quantitatively analyze its position.

While $d_2$ increases both at $z_0$ and at $z_m$, on the lateral sides of the stretched film we see some vertical grey zones and big grey drops breaking horizontality of the grey zone close to $z_0$. Because of the isotropic response to stretching, all the processes are symmetrical and we have vertical grey zones on the lateral sides of the film.

In the last Figure 3.3g we lost isotropy and we have recirculation of all the zones. We define $t_{last}$ as the time after which symmetry is lost (see Figure 3.3h, where the thickness profile is not symmetrical anymore).

For the time $t < t_{last}$ the symmetry is the indication that we are in the horizontal position with $\theta = 0^\circ$ and gravity does not play any role. This fact was used as the experimental method to tune the motorized table to find the reference position.

We stop our analysis at $t_{last}$. 
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(a) The film at \( t_0 = 0 \) sec

(c) The film at \( t_2 = 0.095 \) sec

(e) The film at \( t_3 = 0.4087 \) sec

(g) The film at \( t_4 = 0.9881 \) sec

Figure 3.3: Soap film transformation from \( L_0 \) to \( L_0 + \Delta L; S_4 \) solution. On the left, we see images (a), (c), (e), (g) from the color camera; on the right (b), (d), (f), (h) - corresponding thickness profiles
To summarize and to illustrate the notation we have just introduced above, we consider the film $F_1$ at $t_s < t < t_{\text{last}}$. In the $xz$ (Figure 3.4a) and $yz$-planes (Figure 3.4b) we define $d_1$ and $d_2$. The first is the width of the initial film, $d_2$ is the width of each of the thick films (the Frankel’s films).

![Figure 3.4: Stretched film at time moment $t_s + \Delta t$, where $\Delta t$ is small compared to $t_s$; $d_1$ - width of thin film in the middle; $d_2$ - width of extracted film](image)

Let’s call the velocity of the frontier between thick $(d_2)$ and thin $(d_1)$ films as $V_{fr} = \frac{d(d_2)}{dt}$ - the velocity of the Frankel’s film front, with respect to the meniscus.

### 3.4 Frankel’s film velocity

In this section I would like to explain the technique of thick extracted film velocity determination ($V_{fr}$).

As explained above, for recording the entire soap film we use the color camera described in Paragraph 2.3.2. The camera gives us the image of the entire soap film and we can follow the sharp boundary between $d_1$ and $d_2$. The disadvantage of the color camera is the fact that we have only 80 fps so it provides us only a few frames (around 15) during the stretching process. We may also use the data obtained with the hyperspectral camera with a higher frame rate (296 fps) but the treatment of the data from the hyperspectral camera gives plenty of lost pixels (it will be shown later), which makes the velocity treatment impossible.

First of all, to measure $V_{fr}$, we need a clear representation of $d_2$ evolution in time. Each frame of a video file is an image containing information about space in $xz$-plane and about its colors - intensity RGB. Each parameter (red, green, and blue) defines the intensity of the color as an integer between 0 and 255.

To avoid problems coming from the lateral sides of a film (isotropic response to stretching as shown on Figure 3.3e), we choose the middle of each image ($x = w/2 = 25$ mm). Let’s
Imagine a vertical line at \( x = 25 \) mm. We define as \( z_{\text{border}}(t) \) the position of the border between the thick and the thin parts of the film (see Figure 3.5). At different times the pixel has different positions along the \( z \)-axis. So, to follow this pixel in time, we need to get a spatial-temporal image \((z/\text{frame})\) by making a cut at \( x = 25 \) mm.

An example of a spatio-temporal image is shown in Figure 3.6. This example was reduced up to \( t_{\text{last}} \) to show only processes taking place before recirculation appears. The red pixel with coordinates \((t_1, z_{\text{border}}(t_1))\) represents the same pixel as on Figure 3.5 at \( t_1 = 0.096 \) seconds. We want to find positions for all the pixels \((25, z_{\text{border}}(t))\) on the border.

### 3.4.1 Image processing

In this section I present the technique of boundary determination, which can be afterwards connected to \( V_{fr} \). This technique is different for the two extracted films (at free and supported menisci).

We start the image analysis from an extracted thick film at the free meniscus \( z_0 \). Having a full spatio-temporal image (for all the frames of recorded video file) we need to determine a zone of interest. For this purpose we crop a part of the image as shown by the red rectangle.
on Figure 3.7 for frames $j$ from $j(t_0)$ to $j(t_{last})$, where the boundary behavior is still a straight line. Afterwards, we work only with the cropped part of the image 3.7b.

Each pixel of the cropped image contains RGB information: grey means that we have some average number between 0 and 255 for red, green and blue, for example RGB = (169, 169, 169); bright color (in thin part of a film) means that one of the colors is intensified, for example, if it is blue, it means that RGB has values (0, 0, 255). The main idea for us is to separate thin and thick parts, so, based on the RGB values it is possible to do it automatically.

In order to estimate averaged value for R, G and B in the thick film, we choose a set of pixels (more than 100) and we calculate the average value for $\bar{R}(thick)$, $\bar{G}(thick)$ and $\bar{B}(thick)$.

Next, for each pixel we compute a value based on its RGB and averaged values of RGB. More precisely, we create a new 2D matrix $i \times j$ (the same dimensions as our cropped image), where the value in each box $P(i, j)$ is computed in the following:

$$P(i, j) = \sqrt{(R(i, j) - \bar{R}(thick))^2 + (G(i, j) - \bar{G}(thick))^2 + (B(i, j) - \bar{B}(thick))^2}$$

On Figure 3.8 we see that the values of $P(i, j)$ are laying in the range between 0 and
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Figure 3.7: Spatio-temporal image, $S_4$ solution; (a) - red rectangle is cropped part for $j$ from $j(t_0)$ to $j(t_{last})$; (b) - cropped image

250, where the thick film part has value smaller than 50 while the thin part, close to the boundary, has values around 150. Having this sharp difference between thick $P(i, j)_{\text{thick}}$ and thin $P(i, j)_{\text{thin}}$ parts of the film, we can choose a threshold ($TS$) verifying:

$$P(i, j)_{\text{thick}} < TS < P(i, j)_{\text{thin}}.$$  

For our example we used $TS=55$. Then, if $P(i, j) < TS$, we rewrite it with value "0"; $P(i, j) = 0$. If $P(i, j) > TS$, we rewrite it with value "-1"; $P(i, j) = -1$, so, finally, we have
a binary image, where a boundary between the thick and the thin part of the film is well defined (Figure 3.9). Next, we apply a contour function of Matlab to find all the contour lines between $P(i, j) = -1$ and $P(i, j) = 0$. The function creates a big 2D contour matrix with a list of (X,Y) coordinates for each detected contour. As we see in Figure 3.9 besides the zone corresponding to the thick part of the film, there are $P(i, j) = 0$ in the zone of thin film. That is why the Matlab function finds several contours and we need to choose the right one. The example of cropped image with the contour on it is shown in Figure 3.10a.

![Figure 3.9: Binary image with strong boundary between pixels of thick and thin parts of the film](image)

![Figure 3.10: Different images for contour detection; (a) $S_4$ solution; (b) $S_3$ solution](image)

This method is not working ideally for every experiment. Some values $P(i, j)$ in grey part of image are close to the ones from a bright part and vice versa. An example of a not ideally detected contour is presented in Figure 3.10b. In the following analysis we want to consider
only contour points corresponding to the frontier, so for the last case we do not take into account points in contour which are out of the boundary between thick and thin parts of the film as shown in Figure 3.11 and we suppress the wrong part of the curve by hand.

Figure 3.11: Final contour for images which are not ideal for this analysis

Thus, each point of the found contour defines the distance between \( z_0 \) and the frontier \( z_{\text{border}}(t) \) and we called it \( d_2^{\exp}(t) \):

\[
d_2^{\exp} = z_{\text{border}} - z_0.
\] (3.1)

Next, we consider the extraction of the Frankel’s film from the supported meniscus (meniscus in contact with mobile frame at \( z_m \)) and we present an image processing below.

The technique of image treatment is a bit different. In Figure 3.7a for \( i \) from around 300 to 500 pixels we see that there is extraction of a Frankel’s film from the supported meniscus but the velocity of extraction is less visible while having slope of piezo motor motion. A better visualization is obtained by modifying the image. The idea is to eliminate the motor slope from \((t_0)\) to \((t_0 + t_s)\).

Let’s fix one point \( A(i, j) \) (see Figure 3.12) on the border of the mobile frame. The frame indices corresponding to the period \((t_0, t_s)\) are called \((j_0 + n)\) with \( n \) between 0 and \( n_{\text{max}} \). For each \( n \) in this range I move the pixels \( A(i, j_0 + n) \) at the position \( A(i + SL \cdot n, j_0 + n) \), where we introduced a constant value \( SL \) ("slope"), which corresponds to the amount of pixels (in \( z \)-direction) on which we shifted down our point. As a result we have an image where we do not see the motor movement (Figure 3.13).

The quality of the resulting image does not let us use the method of contour determini-
Figure 3.12: Spatio-temporal image; $S_4$ solution; shows the shift of point $A(i, j)$ during motor motion; for the next pixel $j + 1$ the point is shifted down at value $SL$.

nation so we suppose a constant velocity of the frontier from $t_0$ to $t_s$. The resulting image is presented on Figure 3.13, red line shows the front between thick and thin films.

Figure 3.13: Modified spatio-temporal image; red line shows the front of Frankel’s film extraction.
3.4.2 Hyperspectral camera image treatment

In this section, I would like to explain methods which were done for soap film thickness measurements. All the results provided below are preliminary: we did not have time to exploit all the information provided by the camera and we just extracted the average thick and thin film thicknesses.

As it was explained above in Figure 3.2 with a camera $C_2$ we record the spectral data along the line $L$ with the chosen frame rate (2.3.3).

The hyperspectral camera data were saved as a 3D cube ($z, t, \lambda$). The thickness profile $h(z, t)$ was obtained using expression 4.1.

The Matlab code to analyze data was developed by Antoine Berut and Adrien Bussonnière. Both cameras are synchronized and spatial coordinates can be converted between the two types of images (see 2.3.5). Therefore, we can verify that the data are consistent (by comparing the thickness values from the hyperspectral camera and the entire image of a color camera with color chart 1.9).

It provides an image consisting at each pixel $(i, j)$ information about frame numbers $(j)$, spatial position $(i)$ and pixel values are a soap film thickness $h(i, j)$ (Figure 3.14b). The color code for film thickness $h(i, j)$ is presented on the right of the image from intense blue (zero) to intense yellow (3.5 microns and more). Here free meniscus ($z_0$ on color image) is located at the bottom of the image.

As we can notice, there are points with zero values of thickness on the map. It means that at this point the data are too noisy to extract the thickness. This difficulty leads us to the fact that it is almost impossible to get data for the film thickness during the piezo motor motion. That is why this technique can be used only at $t > t_s$.

As an illustration, the information obtained with both cameras on the same film is shown in Figure 3.14. The thickness map (Figure 3.14b) has been used to determine the average thickness in the initial and Frankel’s film, at a time $t_1 = t_s + \Delta t$, where $\Delta t$ is some experimental value $\Delta t < t_{last} - t_s$. We explain the measure below.

After recording a video file of an experiment, I choose one image from this video when the piezo motor has already stopped at time $t_1 = t_s + \Delta t$ (Figure 3.14a where the white vertical line is the position of $L$).

On the thickness map (3.14b) we identified the line $j$ corresponding to this time $t_1$ (see the dashed line). The cameras were synchronized, so we can be sure to find exactly the same time moment (for this example, $t_1 = t_s + \Delta t$, where $\Delta t = 1.02$ sec). On the color image, I measure the position of the frontier. Using the scaling (1 color pixel = 0.66 spectral pixels)
Figure 3.14: (a) The image from the color camera at $t_1 = t_s + \Delta t$, where $\Delta t = 1.02$ sec; $S_3$ solution; (b) Spatio-temporal image for thickness map. Color code for thickness $h$ is in the right (in microns). Red star - the end of thick Frankel’s film; two green stars limits 5 mm in thin film

and the reference frame positioning explained in Paragraph 2.3.5, I determine the pixel $i_r$ corresponding to this frontier position on the spectral image (see the red star in Figure 3.14b). Then, I select a 5 mm segment in the thin part (thus above $i_r$, see the green stars at $(i_{g1}, j)$ and $(i_{g2}, j)$ in Figure 3.14b). The thickness $h_1$ used in the following is the thickness average along this segment. Similarly, the thickness $h_2$ is the thickness average at time $t_1$, between the meniscus ($z_0$) and the pixel $i_r$.

The results for the example of Figure 3.14 are: for the thick film - 1.36 micron and thin film - 0.59 micron.

To check and to control the reproducibility of the experimental results, we calculate the thickness of the thick and the thin parts by the explained technique for experimental series performed for two different solutions ($S_3$ and $S_4$). Thickness values are presented in Figure 3.15. The thickness of the initial film is kept the same for both types of chemical solutions; for the solution $S_3$ results are well reproducible; for the solution $S_4$ we have some deviation which might be explained by noise in a raw data.
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Figure 3.15: Thickness of extracted Frankel’s film (+) and initial film (○) for $S_3$ (blue) and $S_4$ (red) solutions

3.4.3 Experimental results

Now, we want to interpret the data which we have got from image treatment (see Paragraph 3.4.1).

During a soap film deformation we extract new thicker films from both menisci at $z_0$ and at $z_m$. The width of these films $d_2(t)$ increases faster during the piezo motor motion (from $t_0$ to $t_s$) and slower at $t > t_s$. This can be noticed by plotting the contour of thick/thin film frontier (see Paragraph 3.4.1). However, it does not stop when the motor stops, indicating some elastic behavior of the system.

In Figure 3.16 red stars present each point of the contour extracted on the image (see Figure 3.10a), which is also $d_2$ as a function of $t$. To characterize a velocity of Frankel’s film extraction, we try to fit this line in the best way. From the beginning to $t_s$ the curve is close to a line so we fit it with a polynomial of the first order. So, for $t < t_s$

$$f_1(t) = at + b$$

(3.2)

The second part is more complicated: from $t_s$ to $t_{last}$. We chose the following formula (for $t > t_s$):

$$f_2(t) = a_1(1 - e^{-a_2t}) + a_3t + a_4.$$  

(3.3)

Thus, the fast part is exponential, the slow part is linear. Imposing the continuity of the curve, we connect the two parts (fitting by the first order polynomial 3.2 and by the formula (3.3)) exactly at $t_s$. 

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The result of fitting is presented on Figure 3.16. The blue vertical line shows the separation point \( t_s \). The first part (from \( t_0 \) to \( t_s \)) is fitted with three lines which estimate a standard deviation, the second part is fitted with formula (3.3).

![Figure 3.16: Example of fitting the contour which characterize the position of a front between thick and thin soap films; \( S_3 \) solution, experiment number 2](image)

As we see on Figure 3.16 experimental points are well fitted with chosen rules so we can estimate a value of the front velocity \( V_{fit}^{fr} = \frac{df_{1,2}(t)}{dt} \). For the first part from \( t_0 \) to \( t_s \) we get \( V_{fit,1}^{fr} = a \).

The second part from \( t_s \) to \( t_{last} \) is well fitted by formula 3.3. The coefficient \( a_2 \) is a characteristic time or a time needed for a system to relax. At the big values of \( t \) the velocity of the boundary between a thick and a thin part is defined by the coefficients \( a_3 \) and \( a_4 \).

We needed to introduce a linear term \( a_3 \) in the law 3.3 to improve the fit of some data. In an imaginary "ideal" experiment without any physical effects disturbing the stability, the size of the soap film extracted from the meniscus should reach a constant value \( d_{2,max}^{exp} \) at long time, after motor stops.

The velocity at which we stretch the film is \( U = 80 \text{ mm/sec} \), and \( a_3 \) corresponds to the velocity of the fitted curve \( V_{fit}^{fr} \) at long time. Comparing the data for \( a_3 \) from the image 3.17 and \( U \), we can say that \( a_3 \ll U \), so we can consider \( d_{2,max}^{exp} \) having a constant value at long time. The small variation of \( a_3 \) from zero position might be explained by several factors: not
exact position of contour during image analysis or not correctly chosen time moment $t_{\text{last}}$ (lateral sides recirculation effect influences the contour).

From the experimental data, we estimate $d_{2,\text{max}}^{\text{exp}}$, the asymptotic value of $d_2$, as the size of the extracted film at $z_0$ at the last time $t_{\text{last}}$ just before recirculation appears. To compare these values for all the experiments, we present them on Figure 3.18 for both chemical solutions $S_3$ and $S_4$. For each solution ($S_3$ and $S_4$) the measure of $d_{2,\text{max}}^{\text{exp}}$ is reproducible with an error bar of the order of 20%. Both chemical compositions lead to comparable values, $d_{2,\text{max}}^{\text{exp}}$, being larger for $S_3$ than for $S_4$, but with a difference close to our error bar. An error bar of experimental measurements is a range of one pixel, so for the scaled values of $d_2$ it is around 0.03 mm.

At short time the velocity of extraction is given by $V_{fr}^{\text{fit},1} = a$, where we fitted experimental points by the line at the free and supported menisci. The results for both chemical solutions are presented in Figure 3.19.

The experiments are reproducible with a variation, which indicates our error bar.

The deviation for $a$ at $z_m$ might be bigger than error bar because the method of image treatment is less precise than for thick film extracted at $z_0$. We see that the behavior at the free $z_0$ and supported $z_m$ menisci are identical. This result, which may be considered as natural, will be rediscussed in Chapter 5, where we show that the neighboring films influence the dynamics.

At short time the velocity of extraction $V_{fr}^{\text{fit}} = a$ is around four times smaller than the velocity of the piezo motor $U$. 
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Figure 3.18: The maximal width of extracted film $d_{2,max}^{exp}$ at $z_0$ for $S_3$ (blue) and $S_4$ (red) solutions

Figure 3.19: The values of the experimental fitting parameter $a$ for the free meniscus ($\ast$) and the supported meniscus ($\square$) for $S_3$ (red) and $S_4$ (blue) solutions

We can notice that there is almost no difference in the front velocity for two different chemical solutions. Adding dodecanol we change the rigidity of a film surface, but this factor does not influence the front velocity. There is also no difference between two menisci taking into account the fact that the image processing at $z_m$ is less precise as at $z_0$.

We considered situations at long time and at short time but it is still missing the transient situation. In our experiments we have two transient: just after the motor begins to move at $t_0$, and just after the motor stops at $t_s$. The first transient is characterized by $t_1$, defined as
the extrapolation of the short time fit equation 3.2 at $d_2 = 0$ (see Figure 3.16). So $t_t = -b/a$. This is a transient, which we do not see due to imperfections in experimental techniques and image analysis. The second transient is directly visible in Figure 3.16 and the coefficient $1/a_2$ measures the corresponding relaxation time. Presenting both parameters on the same image (see Figure 3.20) we observe that these values are identical taking into account an error bar. The stars on the image 3.20 present the values of $t_t$ and the diamonds show $1/a_2$ for both chemical solutions.

![Figure 3.20: The values of the experimental fitting parameter $1/a_2$ (○) and $t_t$ (*) for $S_3$ (blue) and $S_4$ (red) solutions](image)

Collecting the effects we observed in the experiments, the initial film obeys the following steps. During the imposed deformation, the total area of the initial film increases. First, we have a transient zone $[0, t_t]$, which is not possible to observe in current experimental conditions, then at short time $[t_t, t_s]$ we have an initial film extension with a fast symmetrical extraction of a new film from both menisci. At the time $t_s$, when the motor has just stopped, we got a new stretched film which is a composition of three parts: two fresh thicker films extracted at $z_0$ and $z_m$ and a thinner soap film in between the extracted ones. Afterward, keeping the film at rest at long time, the extraction of the thicker films continues but at very slow velocity, which we neglect compared to the velocity of stretching $U$. For the maximal size of the extracted film $d_{2, \text{max}}^{exp}$ we expect it to be half the amplitude $\Delta L = 13$ mm. As it is shown in Figure 3.18 for both chemical solutions the values of $d_{2, \text{max}}^{exp}$ are smaller than $\Delta L/2$. The initial film does not recover its size, it behaves as an elastic with memory loss.
3.4.4 Model

In this section, we build a model in order to interpret the dynamics of film extraction shown in Paragraph 3.4.3. We show that the initial film and the extracted films can be modeled as elastic and viscous elements associated in series in the stretched film, as shown in Figure 3.21.

![Diagram of modeled extracted films as elastic and viscous elements](image)

Figure 3.21: Modeled extracted films as elastic and viscous elements

We use the meniscus tension $\gamma$ as a reference and denote by $\sigma$ the tension difference (including surface tension and viscous terms), relatively to this reference. A first remark is that the Laplace pressure, the inertia and the gravity terms are negligible in the flat film. The only force acting on a piece of film is thus the tension, which must be uniform to ensure the force balance. As a consequence, the tension in the initial film is the same as in the Frankel’s film (outside the dynamical meniscus). Using the equation 1.38, we can express the tension in the Frankel’s films as

$$\sigma_{Fr} = 1.85 \gamma (3Ca)^{2/3}$$  \hspace{1cm} (3.4)

with $Ca = \eta V_{Fr}/\gamma$.

This expression is only valid for incompressible interfaces. In this simple model, we assume that the interface extension, that we discuss just after, does not modify significantly the tension induced by the film extraction [49].
3.4. FRANKEL’S FILM VELOCITY

Moreover, in order to build an analytically solvable linear model, we choose to approximate equation 3.4 as

$$\sigma = k \dot{d}_2$$

(3.5)

with $\dot{d}_2 = V_{fr}$,

$$k = 1.85 \gamma \left( \frac{3\eta}{\gamma} \right)^{2/3} \left( \langle V_{fr} \rangle_t \right)^{-1/3},$$

(3.6)

and $\langle V_{fr} \rangle_t$, the average velocity of the Frankel’s film extraction during motor motion, i.e. our fitting parameter $a$ in Figure 3.19. This corresponds to the two viscous elements in Figure 3.21. The tension can also be related to the initial film extension, using a Gibbs elasticity $E$. We thus have

$$\sigma = \frac{E}{d_0} (d_1(t) - d_r),$$

(3.7)

which constitutes the elastic part of the model. Here $d_r$ is the relaxed length of the initial film and $d_0 = L_0$ is the initial film size. The total film size $d(t) = d_1 + 2d_2$ is imposed by the motor motion. So we can re-write equations 3.5 and 3.7 as

$$\dot{d}_1 + \frac{1}{\tau} d_1 = \frac{1}{\tau} d_r + \dot{d},$$

(3.8)

$$\dot{d}_2 + \frac{1}{\tau} d_2 = \frac{1}{2\tau} (d - d_r),$$

(3.9)

where we define $\tau = kd_0/(2E)$. This is the internal time scale of the film, which behaves as a Maxwell fluid. Imposing $d(0) = d_1(0) = d_r$ at $t = 0$ and $d(t) = Ut + d_r$ for $t < t_s$ we get

$$d_2(t) = \frac{Ut}{2} \left( e^{-\frac{t}{\tau}} - 1 + \frac{t}{\tau} \right).$$

(3.10)

At later time, $d(t) = Ut + d_r$ and

$$d_2(t) = \frac{Ut}{2} \left[ \left(1 - e^{\frac{ts}{\tau}} \right) e^{-\frac{t}{\tau}} + \frac{ts}{\tau} \right].$$

(3.11)

This crude model predicts the fact that the Frankel’s film grows linearly with time during the motor motion (see Figure 3.22) and then keeps growing after motor stop. However, the predicted maximal value for $d_{2,\text{max}}$ is always half of the motor displacement, which is not the case experimentally. In order to improve our model, we add that the reference length for $d_1$
relaxes towards its actual value with a second internal time scale $\tau_2$. We thus assume that
\[ \dot{d}_r = \frac{d_1 - d_r}{\tau_2}, \quad (3.12) \]

Using equations 3.8 and 3.12, we get for the reference length $d_r$
\[ \ddot{d}_r + \left( \frac{1}{\tau} + \frac{1}{\tau_2} \right) \dot{d}_r = \frac{1}{\tau_2} \dot{d}, \quad (3.13) \]

We define $\tau_3 = (1/\tau + 1/\tau_2)^{-1}$ and find the solution, for $t \ll t_s$
\[ d_r = \frac{\tau_3^2}{\tau_2} U \left( \frac{t}{\tau_3} + \left( 1 - e^{-\frac{t}{\tau_3}} \right) - 1 \right) + d(0), \quad (3.14) \]
\[ \dot{d}_r = \frac{\tau_3}{\tau_2} U \left( 1 - e^{-\frac{t}{\tau_3}} \right). \quad (3.15) \]

After $t_s$, we get
\[ d_r = \frac{\tau_3^2}{\tau_2} U \left( \frac{t_s}{\tau_3} + \left( 1 - e^{-\frac{t_s}{\tau_3}} \right) e^{-\frac{t}{\tau_3}} \right) + d(0), \quad (3.16) \]
\[ \dot{d}_r = \frac{\tau_3}{\tau_2} U \left( e^{\frac{t_s}{\tau_3}} - 1 \right) e^{-\frac{t}{\tau_3}}. \quad (3.17) \]

We deduce the following predictions:

Before $t_s$
\[ d_1 = d(0) + \frac{\tau_3}{\tau_2} U t + \tau_3 U \left( 1 - \frac{\tau_3}{\tau_2} \right) \left( 1 - e^{-\frac{t}{\tau_3}} \right), \quad (3.18) \]
\[ 2 \ d_2 = U \alpha \left( t - \tau_3 \left( 1 - e^{-\frac{t}{\tau_3}} \right) \right), \quad (3.19) \]
where $\alpha = \frac{\tau_3}{\tau + \tau_2}$.

After $t_s$
\[ d_1 = d(0) + \frac{\tau_3}{\tau_2} U t_s + \tau_3 U \left( 1 - \frac{\tau_3}{\tau_2} \right) \left( e^{\frac{t_s}{\tau_3}} - 1 \right) e^{-\frac{t}{\tau_3}}. \quad (3.20) \]
\[ 2 \ d_2 = U \alpha \left( t_s - \tau_3 \left( e^{\frac{t_s}{\tau_3}} - 1 \right) e^{-\frac{t}{\tau_3}} \right) \]

The illustration of $d_1(t)$, $d_2(t)$ and $d(t)$ are presented in Figure 3.22.

In this simple, linear model, we predict a characteristic time for the transient, both as the motor begins to move or stops, which is $\tau_3$. We can introduce the characteristic time for
3.4. FRANKEL’S FILM VELOCITY

Figure 3.22: $d_1(t), d_2(t), d(t)$; the black straight line, which crosses $d(t)$ at $d_{2,\text{max}}$, is the asymptotic line showing the constant value of $d_2$ at big $t$; the black straight line, which crosses $t$ at $t_t$, is the asymptotic line for an elastic stretching the first part as $t_t^{th}$

$$t_t^{th} = \tau_3. \quad (3.22)$$

If the motor moves for a longer time than $\tau_3$ than we expect a linear variation of the Frankel’s film size with a slope

$$a^{th} = U\alpha/2. \quad (3.23)$$

At long time, the Frankel’s film extension is

$$d_{2,\text{max}}^{th} = Ut_s\alpha/2. \quad (3.24)$$

3.4.5 Comparison Frankel’s theory and experimental thickness

The fresh film thickness must be compared to Frankel’s law (1.37), which quantifies the extraction of a soap film from a bath assuming incompressible interfaces.

To compare the experimental data with theoretical predictions we need to analyze, first
of all, the velocity of the extracted film. In the previous Paragraph (see 3.4.3) we show in Figure 3.16 that velocities during and after motor motion are very different from the motor velocity $U$.

In Figure 3.16 we presented the evolution of $d_2$ with time and fitted the experimental data with two different functions during motor motion and after. From this fit we can get the velocity at which the thicker film is extracted from the meniscus. Next step is to try to correlate the velocity of the extracted film $\dot{d}_2(t) = \frac{d(d_2)}{dt}$ with the theoretical thickness $h_{2}^{\text{th}}$ from the equation 1.37 and to compare with the experimental one $h_{2}^{\text{exp}}$.

First, at time $t \approx t_{\text{last}}$ we produce the thickness profile $h_2(z,t)$ (I do not use "=" here because for some experiments exactly at $t_{\text{last}}$ it is not possible to produce the thickness because of plenty of lost points). The example for $S_4$ solution is presented in Figure 3.23. The line on the image is not continuous because some raw data were not possible to treat to get the thickness. We have two extracted Frankel’s films at $z \approx 4$ mm and $z \approx 15$ mm and the initial film at $z$ between 6 and 12 mm. We are interested in the Frankel’s film at $z \approx 15$ mm because it was extracted from the free meniscus and we discussed the velocity of extraction at the free meniscus (see 3.4.3).

![Figure 3.23: The thickness $h_2$ at $t \approx t_{\text{last}}$ as a function of distance in z-direction; $S_4$ solution](image)

The film at the position $z_1$ (see Figure 3.23) at time $t_1$ has been extracted from the meniscus at time $t_{m1}(z_1)$. Thus, the theoretical prediction for the thickness $h_2^{\text{th}}$ is

$$h_2^{\text{th}}(z_1, t_1) = 2.68 \ast W \ast \left(\frac{\eta \dot{d}_2(t_{m1}(z_1))}{\gamma}\right)^{2/3},$$  \hspace{1cm} (3.25)
where \( W \) is a meniscus size, \( \eta \) - the bulk viscosity, \( \gamma \) - the surface tension. To estimate this value, the only missing parameter is \( \dot{d}_2(t_{m1}(z_1)) \). In order to find it we plot \( \dot{d}_2 \) as a function of time for the same experiment. The example is presented in Figure 3.24. The red line on the image has a strong step at \( t_s \) because of the sharp difference between \( V_{fr}^{fit} \) during and after motor stop.

![Graph of \( \dot{d}_2(t) \) as a function of time]

Figure 3.24: The velocity of extracted film \( \dot{d}_2(t) \) as a function of time \( t \) from the fit presented on the Figure 3.16; \( S_4 \) solution

We have chosen \( z_1 \) to be the position of the highest thickness value, so it corresponds to the extraction at constant velocity \( \dot{d}_2(t_{m1}(z_1)) = 27.46 \text{ mm/sec} \) for considered example. The velocity in the interval from \( t_s \) to \( t_{last} \) does not change a lot and the thickness at \( z_2 \) was extracted during this interval. Because of difficulties with raw data (plenty of lost point for \( h_2(z,t) \)) we can not find the exact value of \( t_{m2}(z_2) \) in Figure 3.24, so we take the average value \( \dot{d}_2(t_{m2}) \approx 2 \text{ mm/sec} \).

During these series of experiments we did not measure \( W \) - the Plateau border size, but we measured it for experiments presented in the Chapter 4 and 5 and we found them to be similar and equal to 0.18 mm.

Then, applying \( W = 0.18 \text{ mm}, \eta \) and \( \gamma \) from the table 2.2, we get \( h_2^{th}(z_1, t_1) \approx 6 \text{ \mu m} \). Comparing to the experimental one in Figure 3.23 we found the value \( h_2^{exp}(z_1) = 3 \text{ \mu m} \), which is twice smaller than expected one.

For the second position \( z_2 \) on the thickness profile, we found \( h_2^{th}(z_2, t_2) = 0.8 \text{ \mu m} \), while the experimental one is 2 \( \mu m \). All the experimental data are collected in the Table 3.1.
Table 3.1: The thickness values from fitting curve and from experimental data

<table>
<thead>
<tr>
<th>Thickness</th>
<th>at $t_{m1}$</th>
<th>at $t_{m2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_{2}^{th}$, µm</td>
<td>6</td>
<td>0.8</td>
</tr>
<tr>
<td>$h_{2}^{exp}$, µm</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

The variation of theoretical values from experimental ones may come from different reasons. The extracted film does not reach the highest predicted thickness, it might be explained by the fact that we have not gotten a steady regime and if we continue extraction at the same velocity, we might get the expected thickness $h_{2}^{exp}$.

A variation also may come from the fact that we have not measured the Plateau border size, we assumed it to be $W = 0.18$ mm, the value taken from another experiments (see Chapter 4 and 5). In these series of experiment the $W$ might be a bit higher and this fact can result in correlation between $h_{2}^{exp}$ and $h_{2}^{th}$.

### 3.4.6 Comparison between model and experiment

In this section I want to compare the suggested model and the experimental data from Paragraph 3.4.3.

The model assumes that at long time we have two expressions 3.23 and 3.24 involving the parameter $\alpha$. The first one we can compare with $d_{2,max}^{exp}$ and the second one can be compared with the experimental slope $a^{exp}$ from the fitting curve 3.2. So let’s express the following parameters:

\[
\alpha^{(1)} = \frac{d_{2,max}^{exp}}{Ut_s/2}.
\]  

(3.26)

\[
\alpha^{(2)} = \frac{a^{exp}}{U/2}.
\]  

(3.27)

As by the model we impose $\alpha$ to be the same in 3.23 and in 3.24, so we compare $\alpha^{(1)}$ and $\alpha^{(2)}$. Both values of $\alpha^{(1),(2)}$ are presented for both chemical solutions in Figure 3.25. By stars we note $\alpha^{(1)}$ and by rhombus we note $\alpha^{(2)}$. The data correspondence for each solution is well, the values are slightly different between both chemical solutions.

For the transient zone we defined $\tau_3$ as $\tau_3 = (1/\tau + 1/\tau_2)^{-1}$, which has to be the same for $t \ll t_s$ (motor begins to move) and for $t > t_s$ (motor stops to move). On the experiment we
Figure 3.25: The values of $\alpha^{(1)}$ (•) and $\alpha^{(2)}$ (○) for $S_3$ (red) and $S_4$ (blue) solutions have in the first interval ($t \ll t_s$):

$$\tau_3^{\text{exp},1} = t_t$$

(3.28)

and relaxation time for the second (3.3)

$$\tau_3^{\text{exp},2} = \frac{1}{a_2}.$$ 

(3.29)

The comparison of these values is presented in Figure 3.20 for both chemical solutions. The values correspond rather well, so the model describes the transient zones.

We compared the parameters of the model with the corresponding parameters on the experiment and we can conclude that it is the right model to describe our experiment. Now we can connect the characteristic times $\tau$ and $\tau_2$ given by the model applying the experimental values to express them. So it gives us an estimation of the physical parameters provided by the model.

Quantitatively speaking, we estimate the value for $\alpha^{(1)}$ and $\alpha^{(2)}$ by averaging the experimental data, similarly, we get $\tau_3^{\text{exp},1} = t_t$ and $\tau_3^{\text{exp},2} = 1/a_2$ (averaging also). The results are in the Table 3.2.
Table 3.2: Experimental parameters $\alpha^{(1),(2)}$ and $\tau_{3^{exp},1,2}$ for $S_3$ and $S_4$ chemical solutions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$S_3$ solution</th>
<th>$S_4$ solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha^{(1)}$</td>
<td>0.7438</td>
<td>0.5901</td>
</tr>
<tr>
<td>$\alpha^{(2)}$</td>
<td>0.6420</td>
<td>0.5074</td>
</tr>
<tr>
<td>$\tau_{3^{exp,1}}$, sec</td>
<td>0.0410</td>
<td>0.0549</td>
</tr>
<tr>
<td>$\tau_{3^{exp,2}}$, sec</td>
<td>0.0642</td>
<td>0.0672</td>
</tr>
</tbody>
</table>

From the definition of $\tau_3$ and $\alpha$ (see the equation 3.19) we get two expressions with two unknown parameters:

\[
\begin{align*}
\frac{1}{\frac{1}{\tau_3} + \frac{1}{\tau_2}} &= \tau_{3^{exp,1,2}} \\
\frac{\tau_2}{\tau + \tau_2} &= \alpha^{(1),(2)}
\end{align*}
\]

Solving these system of equations we get the expressions for $\tau_{exp}$ and $\tau_{2^{exp}}$ as:

\[
\begin{align*}
\tau_{exp} &= \frac{\tau_{3^{exp,1,2}}}{\alpha^{(1),(2)}} \\
\tau_{2^{exp}} &= \frac{\tau_{3^{exp,1,2}}}{1 - \alpha^{(1),(2)}}
\end{align*}
\]

Averaging between two values we get for two chemical solutions:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$S_3$ solution</th>
<th>$S_4$ solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_{exp}$, sec</td>
<td>0.0759</td>
<td>0.1112</td>
</tr>
<tr>
<td>$\tau_{2^{exp}}$, sec</td>
<td>0.1713</td>
<td>0.1353</td>
</tr>
</tbody>
</table>

As we defined above, $\tau = kd_0/(2E)$ is the visco-elastic time scale and together with the expression 3.6 we can estimate $E$ - the Gibbs elasticity. For the $S_3$ chemical solution we have value $E = 1.6 \times 10^{-4}$ N/m and for $S_4$ we have $E = 1.2 \times 10^{-4}$ N/m.

The parameter $\tau_2$ has a physical value of the time needed for the string (here for the film $d_1$, which behaves as a string) not to come back anymore to its initial size.
Chapter 4

Instability of character of Rayleigh–Taylor

4.1 Introduction

The well-known Rayleigh-Taylor instability is the instability between two fluids [53], which occurs when the denser fluid is located above the lighter one. In this situation we have an unstable equilibrium of the system of two fluids which can be easily perturbed by the gravity. The heavier fluid moves downwards by forming fingers. We can say that the interface between both fluids is unstable. An equilibrium can be found again due to surface tension that’s why there is a competition between surface tension and gravity.

As the Rayleigh-Taylor instability develops, the initial perturbation progresses from a linear growth phase into a non-linear growth phase. In the linear phase, equations can be linearized and the amplitude of perturbations is growing exponentially with time. In the non-linear phase, perturbation amplitude is too large for the non-linear terms to be neglected. At this stage we observe "plumes" flowing upwards and "fingers" falling downwards.

Performing an experiment with stretching and compressing soap films we have noticed that a similar effect appears in a thin film having the same density everywhere but different thicknesses. So the density difference of the 3D Rayleigh-Taylor instability is replaced by a thickness difference for soap films.

In this chapter, I would like to explain theoretically and experimentally the effect of the instability which takes place in thin films under stretching. Starting with a model and scaling laws, I continue with a big set of experiments implying variation of several parameters of the system and I finish with numerical comparison between experimental data and a linear model.
All the results lead to the publication of an article (see Appendix A).

4.2 Linear stability analysis

As it was discussed in Chapter 3, the process of soap film pulling occurs in two steps: film extension and new thicker film extraction. We call this extracted film a Frankel’s film. In the previous chapter we considered a situation when pulling is taking place in the horizontal plane, without influence of gravity.

Let’s consider now situations when the gravity plays an important role for the force balance of the system.

4.2.1 Main assumptions

The dynamics of a fluid in a soap film is governed by the Navier-Stokes equation. The velocity field is $\mathbf{V} = \mathbf{V}_i(x, y, z, t)$. The view of a soap film from the lateral side is shown in Figure 4.1. The equation for the $z$-component of the velocity is the following:

$$\rho \frac{\partial V_z}{\partial t} + \rho V_z \frac{\partial V_z}{\partial z} = -\rho g_z - \frac{\partial p}{\partial z} + \eta \frac{\partial^2 V_z}{\partial y^2} \quad (4.1)$$

![Figure 4.1: Scheme of the transition between two parts of the film: thick on the left and thin on the right](image)

The interfacial velocity in the $xz$-plane $\mathbf{v}_i$ leads us to the non-slipping boundary conditions
on the surfaces:

\[ y = 0 : V_z = v_z \]  \hspace{1cm} (4.2)

\[ y = h : V_z = v_z, \]  \hspace{1cm} (4.3)

where \( v_z \) is the velocity at the interface. The fluid is steady and the inertial term vanishes, so

\[ \frac{\partial^2 V_z}{\partial y^2} = \frac{1}{\eta} \left( \rho g_z + \frac{\partial p}{\partial z} \right) \]  \hspace{1cm} (4.4)

Integrating twice and applying boundary conditions we get the equation for the velocity:

\[ V_z = \frac{1}{\eta} \left( \rho g_z + \frac{\partial p}{\partial z} \right) \left( \frac{y^2}{2} - \frac{hy}{2} \right) + v_z \]  \hspace{1cm} (4.5)

Or writing the same equation for \( V_x \), we get in each part of the soap film:

\[ V_i(x, y, z, t) = \frac{1}{\eta} \left( \rho g + \nabla P \right) \left( \frac{y^2}{2} - \frac{h_i y}{2} \right) + v_i(x, z, t). \]  \hspace{1cm} (4.6)

The fluctuations of \( h_1 \) or \( h_2 \) with time and space are much smaller than the thickness difference \( h_2 - h_1 \) so a uniform thickness is assumed in each region. The transition between one region to the other occurs on a typical length \( \delta \) of the order of 100 \( \mu \)m. The pressure gradients are governed by the Laplace pressure and are localized at the frontier between both films. They scale as \( \gamma(h_2 - h_1)/\delta^3 \sim 10^4 \text{ m}^2/\text{s} \), which is leading to velocities of the order of 10 \( \mu \)m/s (they scale as \( V \sim \gamma h^3/\delta^3 \eta \)). The gravity-induced Poiseuille flow scales at most as \( \rho g h^2/\eta \sim 10 \text{ \mu m/s} \). The observed velocities are of the order of 10 mm/s and are thus dominated by the interfacial velocities \( \mathbf{v}_i(x, z, t) \).

In the following, we assume that the velocity is uniform across the film and equal to \( \mathbf{v}_i(x, z, t) \), and that the pressure is homogeneous and equal to the air pressure. One important consequence is that there is no liquid flux across the frontier \( d \) between both film regions.

Assuming an inextensible interface model together with the uniform velocity across the interface, we obtain that an elementary piece of film of volume \( h_i dx dz \), taken either in the thin or in the thick film, is a closed material system which keeps a constant thickness with time.

**Reference state of the film:**

Bottom (1): film of thickness \( h_1 \) between \( z = -d_1 \) and \( z = 0 \) (see Figure 4.2).
Top (2): film of thickness \( h_2 \) between \( z = 0 \) and \( z = d_2 \).
The surface tension gradient balances the weight of the film and the interface is assumed inextensible.

The evolution of the frontier is governed by the gravity $g$, the surface tension of the film, which reference value is $\gamma = 2\gamma_0$ (taking into account both surfaces) and the shear viscosity of the film $\mu_{s,i} = \eta h_i + 2\eta_s$. The interface viscosity is typically of the order of $\eta_s \approx 10^{-6}$ kg s$^{-1}$ [25], much larger than $\eta_h \sim 10^{-9}$ kg s$^{-1}$. The film shear viscosity is thus $\mu_s = 2\eta_s$ both in part (1) and (2). The bulk density is $\rho$.

The instability begins to grow during the Frankel’s film extraction, so the reference state is characterized by $d_1$ and $d_2$ which depend on time. However, in the following, we make the crude assumption that the instability time scale is much smaller than the extraction time scale and that $d_1$ and $d_2$ can be assumed to be constant.

![Figure 4.2: Notations](image)

**Scaling laws for the perturbation**

The boundary between both regions is located at $d(x) = \varepsilon e^{nt} e^{ikx}$. To write a force balance we consider a box of size $\lambda \times \lambda$ as pictured in Figure 4.3a by a red rectangle across the interface, with $\lambda = 2\pi/k$ the perturbation wavelength. The mass of the system is (disregarding any numerical prefactor) $m = \lambda^2 \rho (h_1 + h_2)$. The acceleration is vanishing by symmetry: the left
4.2. LINEAR STABILITY ANALYSIS

Figure 4.3: Various forces acting on system (1) and (2), limited by the red rectangles respectively in the left and right schemes. (a) The force balance on the system (1) involves surface tension and gravity, as viscous forces compensate on both sides. (b) On the system (2), gravity force is reduced by a larger amount of thin film, and interfacial viscous forces and inertia need to be considered.

...part moves up and the right part moves down. The viscous forces are also compensated due to the symmetry of the system.

As the system size in the $z$ direction is of the order of the wavelength, all physical quantities remain invariant in the $x$ direction along the top and bottom boundaries. The surface tension is thus independent of $x$ along these lines and denoted by $(\gamma + \Delta \gamma)$ at the top boundary and $\gamma$ at the bottom.

The gravity force acting downwards can be written as $F_{gr} = m \vec{g} = -\lambda^2 \rho(h_1 + h_2)g$. And finally, we can write force balance due to the surface tension and the weight of the system as following:

$$\lambda \Delta \gamma = \lambda^2 \rho(h_1 + h_2)g, \quad (4.7)$$

$$\Delta \gamma = \lambda \rho(h_1 + h_2)g. \quad (4.8)$$

Now let’s focus on the system schematized in Figure 4.3b, the half of the previous system in
x-direction. The surface tension force is still the same and equal to

$$F_{st} = \lambda(\gamma + \Delta \gamma) - \lambda \gamma,$$  \hspace{1cm} (4.9)

$$F_{st} = \lambda \Delta \gamma.$$  \hspace{1cm} (4.10)

The mass in this system is not balanced by the symmetry anymore and we have to take into account that a thin film is perturbed with the amplitude $\varepsilon$. So, writing the expression for the mass, we have $m_1$ for the thin film and $m_2$ for the thick one:

$$m_1 = \rho \lambda (\lambda + \varepsilon) h_1,$$  \hspace{1cm} (4.11)

$$m_2 = \rho \lambda (\lambda - \varepsilon) h_2,$$  \hspace{1cm} (4.12)

and the total mass is equal to:

$$m = \rho \lambda (h_1 (\lambda + \varepsilon) + h_2 (\lambda - \varepsilon)).$$ \hspace{1cm} (4.13)

The surface tension is slightly overcompensated by the gravity

$$F_g = -\lambda \rho g (h_1 (\lambda + \varepsilon) + h_2 (\lambda - \varepsilon)).$$ \hspace{1cm} (4.14)

The viscous forces are not compensated as well and it acts downwards and gives us (neglecting any prefactor)

$$F_{vis} = -\mu_s \left( \frac{n \varepsilon}{\lambda} \right) \lambda,$$ \hspace{1cm} (4.15)

$$F_{vis} = -\mu_s \rho \varepsilon.$$ \hspace{1cm} (4.16)

In this system we cannot neglect inertia, therefore

$$ma = n^2 \varepsilon \lambda^2 \rho (h_1 + h_2).$$ \hspace{1cm} (4.17)

Finally, we can write the force balance in the system as following:

$$ma = F_{vis} + F_{st} + F_g.$$ \hspace{1cm} (4.18)
4.2 LINEAR STABILITY ANALYSIS

\[ n^2\varepsilon \lambda^2 \rho (h_1 + h_2) = -\mu_s n \varepsilon + \lambda \Delta \gamma - \lambda\rho g (h_1 (\lambda + \varepsilon) + h_2 (\lambda - \varepsilon)). \]  

(4.19)

And using the expression for the surface tension difference from the first case, we can write down final force balance:

\[ n^2\varepsilon \lambda^2 \rho (h_1 + h_2) = -\mu_s n \varepsilon + \lambda^2 \rho g (h_1 + h_2) - \lambda\rho g (h_1 (\lambda + \varepsilon) + h_2 (\lambda - \varepsilon)), \]  

(4.20)

which gives us after transformation the expression:

\[ \lambda^2 \rho (h_1 + h_2) n^2 + \mu_s n - \lambda\rho g (h_2 - h_1) = 0. \]  

(4.21)

At large wavelength, the damping term is the film inertia scaling as \( n^2\varepsilon \lambda^2 \rho (h_1 + h_2) \), and at small wavelength it is the interfacial viscosity.

In the equation 4.21 we have neglected the line tension between the domains (1) - thinner film and (2) - thicker one. The order of magnitude of this tension can be determined from the area excess induced by the frontier (see Figure 4.4). If the thickness varies from \( h_1 \) to \( h_2 \) over a distance \( \delta \) perpendicular to a local line element \( d\ell \), the area of the tilted interface is

\[ a_{int} = d\ell(\delta^2 + (h_2 - h_1)^2)^{1/2} \sim d\ell \delta \left(1 + \frac{(h_2 - h_1)^2}{2\delta^2}\right). \]  

(4.22)

Figure 4.4: Origin of the line tension. The length \( ds \) measured along the interface shape is slightly larger than the distance \( \delta \) along the direction \( e_n \) normal to the frontier. It induces a resulting force \( T \) in the direction of the frontier tangent.

The excess area due to the thickness variation is thus \( (h_2 - h_1)^2/(2\delta) \) per unit length. The resulting line tension scales as

\[ \gamma(h_2 - h_1)^2/\delta \sim 10^{-10} N, \]  

(4.23)
and the associated force $T$ (see Figure 4.3b) on the system scales as

$$-\frac{\varepsilon \gamma (h_2 - h_1)^2}{\lambda \delta}. \quad (4.24)$$

The driving force for the instability is the excess of gravity and three damping and stabilizing terms are the line tension, the interfacial viscous forces and the inertia. The line tension term dominates at very small wavelength. This term ensures the stabilization of perturbations having a wavelength smaller than the critical one $\lambda_c$, at which the destabilizing gravity and surface tension is balanced by the stabilizing line tension:

$$\lambda \rho g \varepsilon (h_2 - h_1) = \frac{\varepsilon \gamma (h_2 - h_1)^2}{\lambda \delta}. \quad (4.25)$$

Leading to the expression for $\lambda_c$:

$$\lambda_c = \sqrt{\frac{\gamma (h_2 - h_1)}{\rho \bar{\delta}}} \sim 10 \mu m. \quad (4.26)$$

This length is smaller than $\delta$, and thus, corresponds to wavelengths at which our model of sharp interface is not valid anymore. In the following, we consider only wavelengths larger than $\delta$ and we neglect the line tension.

**Wavelength and growthrate estimation**

All wavelengths larger than the critical one are unstable. For the big $\lambda$ damping is governed by inertia so we have:

$$\lambda^2 \rho (h_1 + h_2) n^2 = \lambda \rho g (h_2 - h_1), \quad (4.27)$$

and from which we deduce the inertial growth rate:

$$n_{in} = \left( \frac{g (h_2 - h_1)}{\lambda (h_1 + h_2)} \right)^{1/2}. \quad (4.28)$$

For the small $\lambda$ damping is governed by viscosity that’s why we have:

$$\mu_s n = \lambda \rho g (h_2 - h_1), \quad (4.29)$$
and the viscous growth rate scales as:

\[ n_{vis} = \frac{\lambda \rho g (h_2 - h_1)}{\mu_s}. \]  

(4.30)

The most unstable wavelength is obtained for the wavelength verifying \( n_{vis} = n_{in} \). In that case, the observed wavelength \( \lambda_{th,1} \) is given by

\[ \lambda_{th,1} \sim \left( \frac{\mu_s^2}{(h_2^2 - h_1^2) \rho^2 g} \right)^{1/3}, \]  

(4.31)

and grows at the rate

\[ n_{th,1} \sim \left( \frac{(h_2 - h_1)^2 g^2 \rho}{(h_2 + h_1) \mu_s} \right)^{1/3}. \]  

(4.32)

For the set of experimental data, which will be discussed below, we assume \( \mu_s = 10^{-6} \) kg·s\(^{-1} \) [25], we have \( h_2 = 2 \) \( \mu \)m, \( h_1 = 0.5 \) \( \mu \)m and we find \( \lambda_{max} = 2.9 \) mm.

In this example, the obtained wavelength is larger than the soap film width that we usually obtain, so the model does not apply. We need to make a cut off at \( d = \min(d_1, d_2) \) of this scaling. Wavelengths larger than \( d \) can not grow, so if \( \lambda_{th,1} > d \) then the fastest wavelength scales as \( d \). As this wavelength is smaller than the visco-inertial transition, its growth rate is controlled by eq. 4.30. This imposes, for \( \lambda_{th,1} > d \),

\[ \lambda_{th,2} \sim d, \]  

(4.33)

and

\[ n_{th,2} \sim \frac{d \rho g (h_2 - h_1)}{\mu_s}. \]  

(4.34)

The obtained scaling laws are compared with the full numerical resolution of the linear problem which allows to obtain the missing prefactors in equations 4.31 - 4.34, leading finally to the predictions of theoretical fastest wavelength shown in Figure 4.5 for both cases of scaling (at the visco-inertial transition and at the case when we cut scaling at the value \( \lambda_{th,2} \sim d \)). At the end of this chapter, after providing the experimental results, I compare curves which have been obtained for the theoretical predictions and the experimental data points. The full numerical resolution of the problem is out of the scope of this thesis and it might be found in the article presented in Appendix A.

Finally, we can say that there is a direct parallel with the usual Rayleigh - Taylor instability in 3D systems, surface tension playing the role of the pressure, film thickness replacing the density, and line tension replacing the surface tension.
4.3 Experiments description

The experimental study of this type of instability was done for two different set-ups allowing to modify the angle of inclination of the soap film plane. The gravity used in the model becomes $\bar{g} = g \cdot \sin \theta$ (SET1 and SET2). Based on the model explained above, we have several parameters to measure and to adjust: the extracted Frankel’s film size ($d_2$), the Frankel’s and initial film thickness ($h_2$ and $h_1$ respectively), the wavelength ($\lambda$), the growth rate ($n$), the gravity ($g$) due to the angle of inclination and the piezo stage velocity ($U$, velocity of stretching). We use several chemical solutions: $S_1$, $S_2$, $S_3$ and $S_4$. The limits of the laboratory equipment do not let us vary all the parameters on the same set up so we built two different ones and the following explanations will be done separately for two setups and then compared all together.

The first set up is SET1 (see Paragraph 2.1.1 and Figure 2.1). In this situation we modify the angle, so the gravity, and we use two chemical solutions: $S_3$ and $S_4$. The measured parameters in this system are: $h_1$ and $h_2$, $d_2$ as well as the parameters needed to characterize the instability: $\lambda$, $n$. The velocity of stretching ($U$) is kept constant and equal to 80 mm/sec. For a chosen angle of inclination, an experiment was performed three times and for each chemical solution it was repeated twice, so we have 60 measurements for each solution and 120 in total for these series of experiments.

The second set up is SET2 (see Paragraph 2.1.1 and Figure 2.1). The chemical solutions are $S_1$, $S_2$ and $S_4$. As in the previous case, we characterize the instability by getting $\lambda$ and $n$ from experimental data. Here we vary the piezo stage velocity $U$. For each value of the
velocity $U$ we carried out a minimum of three experiments and for each chemical solution we repeated the experiments twice, having in total 126 experimental attempts. The film thickness in this configuration, which we used first, was not measured but we can estimate it using the color map shown in Figure 1.9.

The size of the initial film and the amplitude of stretching are kept the same for both series. The waiting time between film creation and stretching is 15 seconds for SET1 and it is equal to 1 minute for SET2.

All the parameters for both experimental set-ups are collected in Table 4.1.

Table 4.1: Set and measured control parameters for all the experiments, which have been performed to observe the instability. The brackets in the velocity $U$ and the angle of inclination $\theta$ mean that we made an experiment three times minimum for each value inside of the brackets. $\lambda$ and $n$ was calculated afterward, based on the experimental data.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Horizontal case</th>
<th>Vertical case</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial film size ($L_0$)</td>
<td>6 mm</td>
<td>6 mm</td>
</tr>
<tr>
<td>Amplitude ($\Delta L$)</td>
<td>13 mm</td>
<td>13 mm</td>
</tr>
<tr>
<td>Chemical solutions</td>
<td>$S_3, S_4$</td>
<td>$S_1, S_2, S_4$</td>
</tr>
<tr>
<td>Velocity of stretching ($U$)</td>
<td>80 mm/sec</td>
<td>[5, 10, 15, 20, 25, 30, 35, 40, 50, 70, 80, 88, 103, 112] mm/sec</td>
</tr>
<tr>
<td>Angle of the inclination ($\theta$)</td>
<td>[1, 2, 3, 4, 5, 6, 7, 8, 9]$^\circ$</td>
<td>$90^\circ$</td>
</tr>
<tr>
<td>Measured parameters</td>
<td>$h_1, h_2, d_2$</td>
<td>$d_2$</td>
</tr>
</tbody>
</table>
### 4.4 Small angle case

In this section, we perform experiments with the SET1. When the soap film plane is completely horizontal, there is no instability, we observe Frankel’s film extraction from the top and bottom menisci and its behavior described in the Chapter 3. Here we consider the angles of inclination from 1° to 9°. The limited value of 9° has been chosen due to the construction of the setup. For this configuration at bigger angles, it was not possible to create a film. But in this range of angles, it is already possible to see the difference in measured parameters for each value of the angle.

As explained in Paragraph 4.2 the gravity is the driving force for the instability so it is crucial for the case of the not horizontal position of deformable soap film, let’s first see the series of experiments, where we will vary gravity as \( g \cdot \sin \theta \).

The scheme of this set up is presented in Figure 3.2. As it was explained in the previous Chapter, the entire film image is obtained with the color camera \( C_1 \) and we measure the film thickness along the line \( L \) with the hyperspectral camera \( C_2 \).

The color camera (see details in 2.3.2) records with a frame rate of 80 fps. To get enough resolution of the entire film image, the objective with big magnification is used. In this case, we use one with a focal length of 50 mm and a maximum aperture f/1.4; the minimum focus distance is 0.45 m.

The experimental protocol is the same as discussed in Section 3.2 for the zero inclination case with one small difference: at the beginning, we set one constant angle of inclination of laboratory table (see Table 4.1) and perform experimental series by following the protocol in 3.2.

#### 4.4.1 Color camera image treatment

During the experiment, we record 4D matrix of data from the color camera (two spatial coordinates, RGB data, and temporal information - frame number). The parameters we want to get by analyzing the data are the wavelength \( \lambda \), the growth rate \( n \) and the width of an extracted Frankel’s film \( d_2 \). We save three images from the full video file: the first one at \( t_{in0} \) - the last moment before the instability appears, when the frontier is still flat so we can measure \( d_2 \); the second one at time \( t_{in1} = t_{in0} + \Delta t_1 \), where \( \Delta t_1 \) is small (\( \approx 0.04 \) sec), the moment when we see small perturbations of the frontier, so we can measure \( \lambda \); and the third one at \( t_{in2} = t_{in1} + \Delta t_2 \) (\( \Delta t_2 \approx 0.14 \) sec), when the instability developed, it lets us measure \( n \). In this case, \( \Delta t_2 \) is bigger than \( \Delta t_1 \), we still have the same wavelength but with a bigger
4.4. SMALL ANGLE CASE

amplitude. In both cases, $\Delta t_{1,2}$ differs a bit from one experiment to another. The example of two images is shown in Figure 4.6.

![Image of two images showing instability development](image)

Figure 4.6: Instability development for the experiment at $\theta = 9^\circ$ at two times $t_{in1} = 2.07$ sec and $t_{in2} = 2.21$ sec from the $t_0 = 0$ when we start stretching; $S_4$ solution. Magenta points on the left image - positions of maxima to calculate $\lambda$

For $d_2$ determination we open the image file at $t_{in0}$ in ImageJ software and measure the amount of pixels from the top meniscus $z_0$ to the end of Frankel's film and transfer it to millimeters using the scaling, which is 0.033 mm in one pixel.

Next, I want to explain in more details the wavelength determination. We tried to automatize this process by finding the contour between the thick Frankel's film and the thin extended initial film (analogically to image treatment in 3.4.1). But this technique gave us very big error bars, and it was decided to make part of the image processing manually. In Figure 4.6a we see magenta points at the maxima positions of the perturbed boundary. Coordinates of maxima were saved and a wavelength was computed as the average distance between magenta points for each experimental image.

At short time the amplitude is too small to be detected, and at large time some fingers disappear by nonlinear coarsening and others develop. So the time $t_{in1}$ results from a compromise, and error bars on the wavelengths are important. We estimate that we may forget or add one point over typically five, leading to an error of $\pm 20\%$.

Finally, to determine the growth rate we followed few peaks between the two times $t_{in1}$ and $t_{in2}$ and measured the amplitude positions $A_1$ and $A_2$ (see Figure 4.7) relatively to the position of the flat extracted film at $t_{in0}$. Theoretically, we expect an exponential growth as:

$$ A = \alpha e^{nt} \quad (4.35) $$
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Writing $A_1 = \alpha e^{nt_{in1}}$ and $A_2 = \alpha e^{nt_{in2}}$, we get:

\[
n = \frac{\ln\left(\frac{A_2}{A_1}\right)}{t_{in2} - t_{in1}}.
\]  

(4.36)

Figure 4.7: Growth rate definition. At $t_{in0}=1.98$ sec extracted film is flat, at $t_{in1}=2.06$ sec it is perturbed weakly, at $t_{in2}=2.20$ sec - strong perturbation

4.4.2 Hyperspectral camera image treatment

From hyperspectral camera we have 3D set of data as it was discussed in 3.4.2. The method of thickness calculation is the same. We determine the averaged thickness of the extracted Frankel’s film and the initial extended film at time $t_{in0}$, when there are no perturbations.
4.4.3 Experimental results

During this experimental series we modified the angle of inclination $\theta$ and we changed the chemical solution between $S_3$ and $S_4$. Accordingly, we want to see the influence of the gravity on the values of the wavelength $\lambda$ and the growth rate $n$. We want to observe the impact of dodecanol on these dependencies because it usually changes the properties of a surface making it more rigid.

In Figures 4.8a and 4.8b we present experimental data for the wavelength as a function of the effective gravity $(g\sin\theta)$.

![Figure 4.8: Wavelength $\lambda$ as a function of effective gravity $\bar{g}$ for two chemical solutions. Color code presents $d_2$ size in mm](image)

Each point in Figure 4.8 has a color, which corresponds to the size of the extracted Frankel’s film $d_2$ at $t_{in0}$. The color chart is given on the right of each image and its values are in millimeters.

The wavelength slightly depends on the effective gravity, it decreases when the gravity increases. The data are rather dispersed, as expected from our error bar on $\lambda$. We checked that this variability was not correlated to the fluctuations of the system characteristics: neither the film thickness variations, nor the width variations of the thick film can explain the data dispersion (at a given angle it is not correlated with the wavelength, see color chart).

For the chemical solution $S_4$ we have similar results, so the rigidity of a surface does not play a big role in the instability effect.

Next, we plot the growth rate as a function of the effective gravity, taking into account the width of an extracted film $d_2$. The results are presented in Figure 4.9.
4.5 Vertical case

In this section I consider similar experiments but for the case when the angle $\theta$ is equal to $90^\circ$. As we fixed the angle, we vary the velocity of stretching $U$ and the chemical solution.

This experimental configuration was built before the previous case (but it is more logical to explain in this order) so here we don’t have the possibility to measure the thickness with the hyperspectral camera. We can only estimate it from images using the color map (Figure 1.9).

We build the SET2 (see Figure 2.2) and install the light source and the color camera as it is shown in Figure 2.15. All the system of two immobile and one mobile frames can be immersed into the chemical solution as we see in Figure 2.7b.

For this experimental configuration we have a possibility to control the meniscus size, thus, for this purpose we install a long range microscope with a collimated light (see Paragraph 2.3.1).

The entire representation of the experimental set up is done in Figure 4.10b. Red lines show directions of light rays from LED lamp (2) obtained by the camera (1) in the reflection point. Blue lines represent the collimated light rays coming from the light source (3) and...
4.5. VERTICAL CASE

recorded by long range microscope (4). The scheme of lateral view is shown in Figure 4.10a.

![Diagram of experimental setup](image)

Figure 4.10: Scheme of the experimental set up in vertical case. (a) Lateral view on the set up; (b) 1 - color camera; 2 - LED lamp; 3 - collimated light source; 4 - long range microscope

### 4.5.1 Experimental protocol

The experimental protocol in the case of vertical position of deforming soap film is the following:

1. Fixing the constant value of the velocity ($U$) of piezo motor choosing step by step values from the range $[5 \ 10 \ 15 \ 20 \ 25 \ 30 \ 35 \ 40 \ 50 \ 70 \ 80 \ 88 \ 103 \ 112]$ mm/sec.

2. Films creation using the reservoir filled with a chemical solution. Let them drain for 1 minute.

3. Run the color camera.

4. At the time $t_0$ we run the piezo stage and at the same time we take a picture of the Plateau border.

5. At the time $t_s$ the motor stops.

6. Wait for 5 seconds and take the second picture of the Plateau border at the time $t_s + 5$.

7. Wait for 5 seconds and stop the color camera recording at time $t_s + 10$.

8. Repeat three times each experiment (for each velocity value).

9. Perform the entire series of experiments for several chemical solutions (see Table 4.1).
4.5.2 Plateau border size

The meniscus size was controlled only for the case of vertical soap films with a long range microscope. Two images of Plateau border size were done at $t_0$ and at $t_0 + 5$ and they are presented in Figure 4.11 on the left and on the right respectively. On the picture we observe two grey lines and the big black part below. As we use a collimated light, we assume that the Plateau border size is the distance between the upper and the lower boundary of the black and grey parts as it is shown by $W$ in Figure 4.11. In Chapter 1, Paragraph 1.1.4 we used the notation $W_1$ for the Plateau border size in this case (see Figure 1.5, left).

For image treatment we used the following technique: each pixel of the original image has its value for RGB. In the dark part each value of RGB is close to zero (for example, RGB = (0, 5, 2)), in contrast, in the white part each pixel has a value close to maximum (for example, RGB = (255, 255, 255)). The original image is shown in Figure 4.12 (1). First of all, we convert it to "uncolored" image by averaging RGB value for each pixel, like $(PV = \text{pixel value})$

$$PV = \frac{R + B + G}{3}. \quad (4.37)$$

So, now each pixel has only one value - averaged and these values are very different for dark and white parts. A converted image is presented in Figure 4.12 (2), where yellow means average close to 255 and blue - close to zero.

Next, for one chosen vertical line in the middle of the image, we produce a graph: averaged pixel value as a function of its position. As we can see in Figure 4.12 (3) there is a sharp
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decrease of averaged pixel values and the size of this decrease gives us the Plateau border size $W$.

Figure 4.12: The Plateau border size determination. (1) - the original image; (2) - the image with averaged values of RGB in each pixel, color bar shows the value in each pixel; (3) - the graph of pixel position as a function of pixel value; $W$ - the Plateau border size; $S_2$ solution

The measurements were done for two chemical solutions: $S_1$ and $S_2$ for all the velocities from the list in Table 4.1.

In Figure 4.11 we see that $W$ at $t_0$ is bigger than $W$ at $t_s + 5$. During these measurements, we want to verify how the Plateau border size $W$ at $t_s + 5$ differs from one experiment to another.

We notice (see Figure 4.13) that the thickness of the meniscus seems to remain constant around 0.18 mm despite the presence of points smaller or higher than this value. However, we notice some drift, its size has tended to increase and it changed from about 0.17 mm to about 0.19 mm. We know that the velocity of stretching $U$ does not influence this variation because the points are represented by order of experiments we carried out: the first points correspond to small and large velocities and the last points correspond to the intermediate ones. We deduce that taking into account the dispersion, the thickness of the Plateau border size is $(0.18 \pm 0.01)$mm.

4.5.3 Image processing

During this experimental session, we used the software of the camera (IC Capture 2.4) to record video files. The soap film was stretched and several physical effects were observed like thinning of the initial film, extraction of a thicker film from top and bottom menisci, destabilization of the top one. The parameters which are measured during image analysis are the extracted film size ($d_2$), wavelength ($\lambda$), growth rate ($n$) for the destabilized extracted film and the velocity ($V_{fr}$) of the extracted film from the meniscus at $z_0$. 

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After performing experiments, a video was saved in .avi format (the example of the video is available on-line [51]). For following treatment we open it in ImageJ, it lets us to review it frame by frame and to save the needed frames as images for next treatment. Saving the picture at time $t_{in0}$, when a thicker film is already extracted but destabilization has not begun yet, we can measure an extracted film size $d_2$ as shown in Figure 4.14.

The calculation of the velocity of the extracted film ($V_{fr}$) from the meniscus at $z_0$ was made as follows: using ImageJ program features, we make a reslice of all frames of one video along a chosen line to get a spatio-temporal image. Having a picture 4.15 we find coordinates of two points (at time $t_0$ and $t_0 + \Delta t$), which give us the possibility to calculate $V_{fr}$ assuming a linear growth of the Frankel’s film. The quality of an image does not let us calculate it more precisely. An improved technique was presented in Paragraph 3.4.1.
Next, when destabilization appears, we want to measure the wavelength. Saving an image at time $t_{in1}$, we want to calculate the distance between two neighboring peaks (Figure 4.16) at positions $x_n$ and $x_{n+1}$. We look for the boundary between the thicker and the thinner films and then we have to plot this border as a function of its positions and to find its peaks. The technique of boundary determination is the same as it was discussed in Chapter 3.4.1 and it is based on the fact that the thicker film is almost grey and the thinner one is much brighter. All calculations were done in Matlab (very similar to what has been done in Paragraph 3.4.1). For peaks determination we used a standard Matlab function "findpeak", this method is not ideal and not multipurpose, it might be applied only for images with big and well-separated peaks. For the others, treatment has been made manually as explained in Paragraph 4.4.1.

$$\lambda = \langle x_{n+1} - x_n \rangle$$

Figure 4.16: Extraction of the frontier by image processing at $t_{in1}$; $S_2$ solution

### 4.5.4 Instability. Physical control parameters

Now let us first provide the results for a velocity of an extracted Frankel’s film $V_{fr}$ and a Frankel’s film size $d_2$.

In these series of experiments we vary the piezo motor velocity $U$, so this is our experimental control parameter. Based on the model, which we discuss in Section 4.2, the physical
control parameters for the instability are $d_2$ and thicknesses $h_1$ and $h_2$. First of all, it is needed to determine the relationship between the chosen experimental control parameters and physical ones. $d_2$ is measured with a good precision (one pixel is 0.048 mm) at $t_{in0}$. In Figure 4.17, we see that with increasing the motor velocity $U$, $d_2$ is increasing from 1 to 2 mm. It gives us information that by changing $U$ we can tune $d_2$, and, in turn, we can control this physical parameter, which is responsible for instability and for $\lambda$ according to the model.

![Graph showing the relationship between $d_2$ and motor velocity $U$.](image)

Figure 4.17: The width of the extracted film $d_2$ at $t_{in0}$ as a function of motor velocity $U$; $S_2$ solution

Other physical control parameters for the instability predicted by a model are thicknesses $h_1$ and $h_2$. They were not controlled in the experiment directly but we can estimate it by two different ways. We tried to get an estimation of $h_2$ by using the color map 1.9 when some green and pink fringes were visible. We also tried to use the Frankel’s film extraction velocity to deduce its thickness, but as shown in Paragraph 3.4.5, this does not lead to a quantitative prediction, given our experimental noise. However, we discuss below the obtained velocity.

To determine the velocity from the image in Figure 4.15, we only have to find the slope of the frontier between the thin and thick parts of the film. The measured velocity is in the range from 1 to 16 mm/sec and it is presented as a function of the piezo motor velocity $U$ in Figure 4.18 for the chemical solution $S_2$. As for the case in Paragraph 3.4.3, the $V_{fr}$ is
4.5. VERTICAL CASE

always smaller than $U$ for the extraction in vertical case also.

![Figure 4.18: The front velocity $V_{fr}$ as a function of the motor velocity $U$ for $S_2$ solution](image)

Determination of this velocity permits us to estimate a value of the thickness $h_2$ with the formula 1.37. In our case the values of $\gamma$ and $\eta$ are taken from the Table 4.1 and the meniscus size $W$ is 0.18 mm.

Taking into account this law we find that the film created from the meniscus at $z_0$ has a thickness between 0.5 $\mu$m for the velocity $V_{fr} = 0.9$ mm/sec and 3.27 $\mu$m for $V_{fr} = 16.2$ mm/sec. The data for high velocities are in correspondence with the thickness, which we can estimate by the color map from Figure 1.9. At slow velocities, we extracted a film, which is slightly thicker than the initial film of thickness $h_1$.

Analogically, we find $V_{fr}$ as a function of $U$ for two other chemical solutions. For the solution $S_1$, the velocity $V_{fr}$ is laying in the range between 1 and 12 mm/sec and this corresponds to a thickness in the range from 0.46 $\mu$m to 2.4 $\mu$m.

For the chemical solution $S_4$ we have the range for extracted film velocity $V_{fr}$ from 2 to 27 mm/sec and taking into account the formula 1.37, calculated thickness has to be between 0.7 $\mu$m and 4.5 $\mu$m.

All the experimental points we can see in Figure 4.19 for $S_1$ solution on the left and the
$S_4$ solution on the right.

![Figure 4.19](image)

Figure 4.19: The front velocity $V_{fr}$ as a function of motor velocity $U$ for $S_1$ solution and $S_4$ solution

The technique of $V_{fr}$ determination is not ideal and it may lead to a big deviation in $h_2$ from its real value. As $d_2$ is very small for slow velocities, it is impossible to estimate the value of $h_2$ by the color map 1.9. Thus, the results of slow velocities are not reliable. This technique was improved in Section 4.4 when we used the hyperspectral camera for thickness measurements.

The values of $h_1$ can be estimated by using the color map 1.9 with an uncertainty of $\pm 100$ nm and for all the velocities it is in the range between $0.2 \, \mu m$ for high velocities and $0.5 \, \mu m$ for slow ones.

To conclude, we demonstrated that we can control the physical parameter $d_2$ by tuning the experimental parameter $U$. As for the thickness difference $(h_2 - h_1)$, which is one of the parameters influencing the wavelength $\lambda$, we can only estimate its value.

**Influence of the solution injection**

The experimental protocol implies time delay between a soap film creation and the moment when the piezo motor starts to move. This delay is set the same for all the experiments but it still does not promise us that at the time $t_0$ (the motor starts to move) all the films are absolutely identical. Trying to improve this problem we install the system of soap solution injection to the film using four syringes explained in the Paragraph 2.1.4. They pump the solution into the film with flux $Q = 0.06 \, ml/min$ each of them. We expect the thickness of the soap film to be more reproducible, than for the case without injection.
4.5. VERTICAL CASE

I performed one experimental series with the $S_4$ solution. The value of $Q$ was chosen equal to 0.06 ml/min to reach the same Plateau border size (around 0.18 mm) as for the experimental series with $S_2$ solution but without infusion. The key parameter for the wavelength is the extracted film size $d_2$, so for the experiments with infusion we repeat the same ones as for the series without infusion (the same range of velocities, see the Table 4.1) and then we compare the values of $d_2$ as a function of the stretching velocity $U$ for both experimental series. The results are provided in Figure 4.20. The blue points in Figure mean the experimental series without infusion, the red ones - with the infusion. As we can notice, more reproducible initial film thickness does not influence the extracted film size that is why the fluctuations in the initial film thickness during experiments can be neglected.

![Figure 4.20: The extracted film size $d_2$ as a function of the motor velocity $U$ for $S_4$ solution. Red points - with infusion, blue points - without infusion](image)

4.5.5 Instability. Results. Wavelength

Now we want to present the results for the wavelength $\lambda$ and for the growth rate $n$ for three different chemical solutions. As it was discussed above, our experimental control parameter is the piezo motor velocity $U$, so, let’s try to see variation of $\lambda$ with $U$ tuning. The method of $\lambda$ determination was discussed above: for $S_2$ solution it was peaks determination on the contour, for others ($S_1$ and $S_4$) it was done manually.

The biggest set of the experiments was done for $S_2$ solution (6 experiments for each value of the motor velocity). The results for $\lambda$ as a function of $U$ are provided in Figure 4.21.
We can notice that for $U < 40 \text{ mm/sec}$ there is a significant increase of $\lambda$ with increasing of $U$. To make this trend more visible, I also traced the average between 6 experiments for wavelengths as a function of the velocity of the motor $U$ in Figure 4.22.
The analysis shows that $\lambda$ increases for $U < 40 \text{ mm/sec}$: from around 1.22 mm to about 1.735 mm. However, this increase of the wavelength seems to reach a plateau from about 40 mm/sec, it saturates at about 1.77 mm. For $U > 40 \text{ mm/sec}$, $\lambda$ is between 1.75 and 1.8 mm. The same tendency we saw for our physical control parameter $d_2$ in Figure 4.17: reaching a plateau for $U > 40 \text{ mm/sec}$.

In order to see the influence of the extracted film size $d_2$ to the wavelength $\lambda$ we plot these variables altogether for all the velocities (Figure 4.23).

![Figure 4.23: The wavelength $\lambda$ as a function of $d_2$ for $S_2$ solution](image)

Looking at Figure 4.23 we can confidently say that bigger $d_2$ provides bigger $\lambda$. So, the extracted film size $d_2$ controls the wavelength of the instability. Note that the equation 4.31 predicts that lambda should decrease with $h_2$. As previously said, the thickness $h_2$ is not measurable for these series, but, if it significantly varies, it should increase with $U$. This would lead to a decrease of $\lambda$ with $U$, in contradiction with the observation.

For two other chemical solutions $S_1$ and $S_4$ we want to observe the same tendency. The results for the wavelength as a function of the motor velocity $U$ and as a function of $d_2$ are provided in Figure 4.24 for $S_1$ and in Figure 4.25 for $S_4$ solutions.

As we can see from the provided pictures for the motor velocities $U$ slower than 40 mm/sec the wavelength increases very fast and then saturates at the value around 2 mm for all tested chemical solutions. It happens when the value of $d_2$ is around 0.7 mm, so for bigger values of $d_2$ the wavelength $\lambda$ is still increasing but much slighter.

The size of extracted film $d_2$ is one of the key parameters for the instability, its increase
Figure 4.24: The wavelength $\lambda$ as a function of (a) the motor velocity $U$ and (b) of the extracted film size $d_2$; $S_1$ solution

Figure 4.25: The wavelength $\lambda$ as a function of (a) the motor velocity $U$ and (b) of the extracted film size $d_2$; $S_4$ solution

leads to higher values of $\lambda$. But we can not say that it is only one parameter which is important. If we come back to the results for small angle situation (see Paragraph 4.4.3), there is a decrease of wavelength, which can not be explained by decreasing $d_2$ (see color bar in Figures 4.8). In that case more important parameters are thickness variation and the effective gravity.
4.5.6 Instability. Results. Growth rate

The last parameter which we can obtain from the experimental data is the growth rate \( n \) of the instability. Considering \( d_2 \) as a control physical parameter, we plot the growth rate as a function of extracted film size \( d_2 \) for two of three chemical solutions: \( S_2 \) and \( S_4 \). Unfortunately, for the experimental series with \( S_1 \) solution the data about time for each frame were lost, that is why the determination of growth rate is not possible. The results we can see in Figure 4.26a for \( S_2 \) solution and in Figure 4.26b for \( S_4 \) solutions.

![Figure 4.26: The growth rate \( n \) as a function of the extracted film size \( d_2 \)](image)

The result for \( S_2 \) solution is a dispersion values of \( n \) between 0 and 35 \( s^{-1} \) (see Figure 4.26a) with no visible trend according to the \( d_2 \) film size, the growth rate does not seem to vary depending on \( d_2 \). According to the model, which might influence the growth rate is difference of thicknesses \( (h_2 - h_1) \), which is another control parameter.

For the \( S_4 \) solution we have the data dispersed from 0 to 25 \( s^{-1} \) and this set of data slightly depends on the extracted film size \( d_2 \). It increases with increasing of \( d_2 \). But the data are still strongly dispersed. The difference between these two cases might be explained by the fact that for the \( S_2 \) solution we carried out much more experiments and tested bigger variation of the velocities of stretching.

Comparing to the data for the SET1 (see Paragraph 4.4.3 and color bar in Figure 4.9b) for which the growth rate is rather dispersed as a function of \( d_2 \) either, we can say that the key factor which has an influence on the growth rate is the gravity in the system.
4.6 Comparison between experimental data and theory

In this chapter I want to compare the experimental data for SET1 and SET2 with the linear analysis which I’ve explained in Section 4.2. The most unstable wavelength is obtained for the case when the viscous term is equal to the inertial one as it was shown in the equations 4.31 and 4.32. To fit the experimental data with this linear model we have to take into account prefactors, which are necessary for this action. The Matlab code for the linear analysis and for this fitting was done not by me, so I won’t provide any details about it, only the final results and the values of prefactors for each experiment.

For the case of small angle position of the soap film (SET1) we provide the theoretical curves for all the experiments and for the vertical position of the soap film (SET2) we plot the theoretical curves only for $S_4$ solution because it is the only common chemical solution for all the experiments (SET1 and SET2).

To compare our predictions with the experimental data, we took into account in the model, for each experimental data, the measured value of the parameters $h_1, h_2, d_2, \bar{g}$. The only unknown physical parameter is the interfacial shear viscosity $\mu_s$. Its value has been set to $\mu_s = 0.8 \cdot 10^{-7}$ kg s$^{-1}$ for both solutions, the order of magnitude of this viscosity is the expected one for the SDS solution. The thickness $h_2 = 1.5 \, \mu$m has been used in the model for the SET2.

Behavior in the linear regime for solution $S_3$ is shown in Figure 4.27, where we have the wavelength $\lambda$ and the growth rate $n$ of the instability as functions of the effective gravity $\bar{g}$. The solid lines are the scaling laws given respectively by equation 4.31 with the prefactor 3 and by equation 4.32 with the prefactor 0.3. The color chart on the right shows the size of the extracted film $d_2$.

The same behavior in the linear regime for $S_4$ solution is provided in Figure 4.28, where we have the wavelength $\lambda$ and the growth rate $n$ of the instability as functions of the effective gravity $\bar{g}$. The solid lines are scaling laws given respectively by equation 4.31 with the prefactor 4.5 and by equation 4.32 with the prefactor 0.25. The color chart on the right shows the size of the extracted film $d_2$.

Experimental wavelength $\lambda$ and growth rate $n$ as functions of $d_2$ for solution $S_4$ with the SET2 are presented in Figures 4.29a and 4.29b respectively. The thin film thickness $h_1$, expressed in $\mu$m is given by the color map 1.9. The dashed lines (at small $d_2$) are respectively the scaling law in the equation 4.33 with the prefactor 0.1 and the scaling law in the equation 4.34 with the prefactor 3.8; the solid lines (at large $d_2$) are scaling law in the equation 4.31.
4.6. COMPARISON BETWEEN EXPERIMENTAL DATA AND THEORY

Figure 4.27: (a) $\lambda$ as a function of the $\tilde{g}$, prefactor for scaling law is 3; (b) $n$ as a function of the $\tilde{g}$, prefactor for scaling law is 0.3; $S_3$ solution; SET1

Figure 4.28: (a) $\lambda$ as a function of the $\tilde{g}$, prefactor for scaling law is 4.5; (b) $n$ as a function of the $\tilde{g}$, prefactor for scaling law is 0.25; $S_4$ solution, SET1

with the prefactor 3.12 and the scaling law in the equation 4.32 with the prefactor 0.18.

First of all, on the curves 4.27a and 4.28a as we noticed before, there is no correlation between $\lambda$ and $d_2$, so the scaling for large $d_2$ is used. The wavelengths in Figure 4.29a are compatible with a scaling as $d_2$ at small $d_2$ and as constant value at large $d_2$.

For the SET1 experiments we see very well correspondence between the theoretical curves and the experimental data. For the SET2 the correspondence is good for the small values of $d_2$, when we are in a viscous regime. However, the prefactors used for these fits differs from the predicted ones. Especially, the power law in 4.31 is predicted to have a prefactor
Figure 4.29: (a) $\lambda$ as a function of $d_2$, prefactor for the scaling law is 3.12 for solid line (at large $d_2$) and 3.8 for dashed line (at small $d_2$); (b) $n$ as a function of $d_2$, prefactor for the scaling law is 3.8 for solid line (at large $d_2$) and for dashed line 0.1 (at small $d_2$); $S_4$ solution, SET2

of the order of 20 (obtained by comparison with the numerical prediction, see Appendix A), whereas a prefactor of the order of 4 is systematically needed to fit the data of Figures 4.27a, 4.28a, 4.29a.

A qualitative agreement is obtained for the growth rate $n$, despite the large dispersion of the data, that reaches a factor 2 in Figure 4.29b, where we have the prefactor for the scaling law 0.18 (the solid line in Figure 4.29b) instead of the expected 0.35.

Finally, we want to make the comparison between the measured wavelength and predicted by the theory for the corresponding parameters (see Figure 4.30). Here the color chart shows the effective gravity $\bar{g}$ for Figure 4.30a and the value of $d_2$ for Figure 4.30b. We obtain a systematic overestimation of the wavelength by the theory, which can not be attributed to the uncertainty on the film thicknesses or on the interfacial viscosity, because their predicted exponents are small. As shown in Figure 4.30, the wavelength depends on gravity, whereas the model predicts it should depend mainly on $d_2$.

The different factors may influence the observed discrepancy. The system is not steady, so the extracted film size $d_2$ changes during the instability development. The observed wavelength may thus be the fastest mode associated with an earlier value of $d_2$, which is smaller as $\lambda_{th}$ increases with $d_2$.

Another possibility is that the boundary conditions at the top used in the model (a vanishing velocity, see the article in Appendix A, part D), are not the actual ones. The normal velocity at the top meniscus should be small, but the tangential velocity at the
Figure 4.30: Theoretical wavelength $\lambda^{th}$ as a function of the experimental value $\lambda^{exp}$ shown in Figures 4.27 (○), 4.28 (●) and 4.29 (●); (a) the color chart shows the effective gravity value $\bar{g}$ in m$^2$/sec, red corresponding to 9.8 m$^2$/sec; (b) the color chart shows the Frankel’s film width $d_2$ in mm.

meniscus may be nonnegligible.

4.7 Conclusion

In this chapter I described the Rayleigh-Taylor instability, which is taken place in a soap film when a thicker part is on the top of a thinner one. This situation is driven by an extension of an initial non-horizontal film, so we can suppose that it should happen in a 3D foam when local films are deformed by coarsening or external shear. This process may potentially have an influence on the thickness distribution in 3D foam, thus it may change the gas diffusion from one bubble to the other and so the coarsening rate, as well as the energy dissipation driven by film shearing and thus the apparent viscosity.

This problem is a good example of 2D Rayleigh-Taylor instability but here we observed in the details only linear evolution; non-linear one requires deeper study.
Part III

Five films set-up
Chapter 5

Five films case

5.1 Introduction

In this part of my thesis, we make a model of foam at a millimetric scale and create a situation of four connected bubbles which produce five thin films between them with two common menisci. Deforming this "foam prototype" we can have an idea of its response to the shear and to relate it to the effective viscosity. This model obeys the Plateau laws (see Paragraph 1.1.4) and requires a new set up to be developed. This device has to allow us to perform several configurations of deformation while controlling independently the size of each film. Also, to analyze the soap film behavior during and after stretching or compressing, film thickness measurements have to be performed and this can answer the question about interfacial stresses.

To visualize the considered model let’s see the scheme in Figure 5.1 in $yz$-plane. Four lateral films "1" and one horizontal film "2" are connected through common menisci "3" of size $W$. The idea is to create a well-controlled dynamical regime at which we can deform all lateral films by stretching or compressing, while the horizontal one stays at rest during deformation. The characteristic dimension of the system in Figure 5.1 is $L = 6$ mm.

An important point of the suggested geometry is that the area of each connected component of the interface might be kept constant during deformation of the films if we pull two lateral films on a diagonal and compress the two films on the other diagonal by the same amount. It can be referred to a 3D sample where the bubble’s area does not change much during deformation but the area of individual soap films is modified. During the imposed deformation, the system can shear the central film without stretching or compressing the interface or stretch (compress) lateral parts while the central film is at rest. This specific
deformation will be investigated in the future but explains the peculiar shape we designed.

When the size of the film is changed, the liquid flows, the film thickness changes. Measuring the thickness we try to answer the question about the nature of the flow. The other important quantities for the flow nature as velocity and concentration fields are out of the scope of this thesis, they were investigated by Adrien Bussonière and the first results can be found in the article, which I co-authored. ([10], see Appendix B).

During one year of my thesis I developed an experimental set up based on the explained geometry and performed the first measurements of a film thickness trying to answer the first but the key question for this geometry: do the neighboring films influence each other during the imposed deformation? The wider question about shearing or stretching the interfaces is still open.

5.2 Experimental device

In order to reproduce on a few thin films the deformations which are taken place in a foam sample, the most important part of developing is a device design. During the first year of my thesis, I created and tested several configurations of the set-up. The last version is presented in Figure 5.2 (called SET3 in the following) and it was built this way to get several advantages which I explain below.

First of all, we consider in Figure 5.3 representation of five connected films ($F_1, F_2, F_3, F_4$ - the lateral films; $F_5$ - the central horizontal film) through common menisci. To create a soap film we need to have two immobile frame pieces (red on the image) $P_1$ and four mobile ones

Figure 5.1: Soap film from the lateral side. "1": lateral films; "2": the central film; "3": Plateau border; L is characteristic size of the system, W is a plateau border size.
5.2. EXPERIMENTAL DEVICE

Figure 5.2: The SET3; (a) the view from the left; (b) the front view. $M$ - linear piezo stages; $P_1$, $P_2$ and $P_3$ are the plastic pieces to create a film; $F_2$ and $F_3$ are the positions of soap films.

- two $P_2$ for top lateral films and two $P_3$ for two bottom films. The film $F_5$ is a horizontal film, which is not deformed during experiments. Two top films $F_1$ and $F_3$ with two bottom films $F_2$ and $F_4$ are stretched or compressed during experiments, using mobile frames $P_2$ and $P_3$.

Figure 5.3: Representation of five connected films. $F_1$, $F_2$, $F_3$, $F_4$ - the lateral films; $F_5$ - the central horizontal film; $P_1$ - immobile frames; $P_2$, $P_3$ - mobile frames

To reproduce two immobile frames $P_1$ the piece shown in Figure 5.4 was printed using 3D printer. It consists of four small wings on which we will hold $F_1$, $F_2$, $F_3$, $F_4$ and it has distance between wings to create and to hold $F_5$. We print two pieces $P_1$ and we install it in front of each other at the distance $w = 40$ mm equal to the length of the menisci. In Figure 5.2a we see them screwed to the metallic feet in the middle of the image. Each wing has
sectional hexagon shape to avoid problems with attaching a soap film to frames (discussed in Paragraph 2.1.1, Figure 2.5).

![Figure 5.4: The piece $P_1$](image)

Next, we need to deform two lateral top films $F_1$ and $F_3$, for this purpose we create the pieces $P_2$ as pictured in Figure 5.5. The long rectangular part is attached to a piezo motor $M$, the piece $P_2$ has the big parallelepiped $P_{inj}$ (for chemical solution injection), the left edge of the piece $P_2$ has sectional hexagon shape for the same reason as in the case of the $P_1$.

Parts $P_2$ have a special pieces $P_{inj}$ for needles (see Figure 5.5). This allows us to inject an extra chemical solution directly to a soap film and to control the Plateau border size together with film thickness.

![Figure 5.5: The piece $P_2$; $M$ is the piezo motor](image)

Finally, the deformation of bottom lateral films $F_2$ and $F_4$ is made with pieces $P_3$ from the Image 5.6. The right part of the frame is attached to $P_1$, the left part is attached to a piezo motor $M$. As the $F_2$ and $F_4$ are inclined at $60^\circ$ from the $z$-direction, in Figure 5.6 the part attached to a motor and the part attached to an immobile frame are inclined either in order to allow for a translation in the plane of the film. As the motor needs to be above the
whole system, in order to remain out of the solution during the film production, a vertical piece is used to transmit the translational motion of the motor to the mobile frame.

The global representation of the set up is presented in Figures 5.2a and 5.2b. All the pieces for the SET3 are made of plastic using 3D printing (see Paragraph 2.1.2).

The construction of the SET3 is easily transformable, so we can modify it if we need to add some complementary devices for measurements. As well as the system is "light", means it is not heavy, so there are a space and access for several optical devices as the color camera, the hyperspectral camera, the long-distance microscope for Plateau border size measurements and there is access at the top for detecting what happens in the horizontal film.

Design and development of the SET3 was a long process but the result has plenty of advantages and gives us the opportunity to perform experimental measurements and to study the influence soap films at each other as in a foam sample. The design of this set up is unique that is why we have got the patent for the invention and called it "Rheometer for liquid films".

For all the experiments below I use the chemical solutions $S_2$ and also for several experiments - $S_1$.

5.3 Static conditions

5.3.1 Plateau border. Measurements

We have discussed the possibility of chemical solution injection in the system. First of all, we want to understand the influence of the amount of injected liquid on the Plateau border
size and to find the best value for experiments. If an infusion is too small, the soap film drains fast and we can not predict its reproducibility from one experiment to another one. If an infusion is too big, the soap film becomes heavy and it deforms because of the gravity.

In this section, I present an experimental data of Plateau border measurements in static conditions. First, I present the technique to measure the Plateau border size. It is very close to what I explained in Paragraph 4.5.2 but here we use another optical device and another angle of observation (see Paragraph 1.1.4, Figure 1.5). In this experimental series, we use a photo camera with special bellows to get magnification 5.25x. The camera was set along the axis $y$, the light source was behind the experimental setup. A schematic representation of the system is shown in Figure 5.7. An example of the typical picture we obtained is shown in Figure 2.13b, where $W$ is the Plateau border width.

![Schematic representation of system.](image)

Figure 5.7: Schematic representation of system. $F_3$, $F_4$, $F_5$ - the thin films; "PB" - the Plateau border

To get the Plateau border width $W$, we measure the distance between two grey lines on the image 5.7. The image is a bit different from what we get with the long range microscope (see the example in Figure 4.11) because here we do not use a collimated light source, we use a LED lamp and the angle of observation is different (see Figure 1.5). The injection to the soap film is done by using a syringe driver explained in Paragraph 2.1.4. Four needles (two for $F_1$ and two for $F_3$) are injecting a chemical solution with chosen flux $Q$. We expect $W$ to grow with increasing flux value of $Q$. We start measurements at $Q_0 = 0$ and we increase it to the value $Q_{big} = 0.05 \text{ ml/min}$. This number for $Q_{big}$ was obtained experimentally: for $Q > Q_{big}$ the system becomes heavy and we notice deformation of the Plateau border from the horizontal position. For each value of $Q$ we need to find time $t_{constant}$, when $W$ does not grow anymore.
5.3.2 Plateau border. Experimental protocol

Observation of the Plateau border size $W$ was done with the following protocol:

1. First, we create a soap film using a reservoir filled with a chemical solution ($S_2$).

2. Next, we run the injection at one constant value $Q$ from the range $[0 \ 0.005 \ 0.01 \ 0.02 \ 0.03 \ 0.04 \ 0.05]$ ml/min through four needles into $F_1$ and $F_3$ at time $t_0 = 0$ sec.

3. We take a picture of the Plateau border at $t_1 = 25$ sec.

4. We continue injection and we take a picture of the Plateau border (of the same soap film) at $t_2 = t_0 + 60$ sec, $t_3 = t_0 + 120$ sec, $t_4 = t_0 + 180$ sec, $t_5 = t_0 + 240$ sec.

5. Collect the data for one chosen value of $Q$ and repeat everything for the next value of flux.

5.3.3 Plateau border. Experimental results

The experimental data we got for each value of flux $Q$. The image treatment was done manually in ImageJ by measuring the number of pixels between two grey lines. In Figure 5.8 we show the variation of the Plateau border width $W$ as a function of time, for different values of $Q$. We noticed that the Plateau border width stabilizes at $t_{\text{constant}} = 100$ seconds for all the values except $Q_0$ because we expect Plateau border size constantly decreases until the film rupture in this case with no injection.

Finally, to qualify and better present the Plateau border width $W$ for each $Q$, we plot $W$ at $t_5 = t_0 + 240$ sec as a function of $Q$. The results are provided in Figure 5.9. For all our following measurements we use $Q = 0.05$ ml/min.

5.4 Dynamic conditions. Neighbouring film influence

5.4.1 Description of experiment

The main advantage of the experiment with five connected films is to allow observation of the influence of neighbouring films on the behavior of the observed one. The idea is to try to answer the question of liquid transfer from one film to another during deformation through menisci.
CHAPTER 5. FIVE FILMS CASE

Figure 5.8: Plateau border width $W$ as a function of time

Figure 5.9: Width of Plateau border $W$ as a function of flux $Q$
5.4. DYNAMIC CONDITIONS. NEIGHBOURING FILM INFLUENCE

In these experimental series, we want to stretch and to compress three films $F_1$, $F_2$ and $F_4$ in different configurations, to observe $F_3$ which is always stretched.

We will measure the extension of $F_3$ as a function of time, while the motor moves and just afterward, from the thickness measurements. We want to see if the extension changes when neighbouring films $F_1$, $F_2$ and $F_4$ are pulled or pushed. The construction of the SET3 is presented in Figure 5.10, where we have the hyperspectral camera $C_2$ (see description 2.3.3) together with the light source (the incandescent lamp) $L_2$ (we use the same notations as in Part II for the SET1 and SET2).

![Figure 5.10: Scheme of the experimental SET3; $F_1$, $F_2$, $F_4$ - lateral films; $F_3$ - the observed lateral film; $F_5$ - the central film; $L_0$ - the initial size of the film; $\Delta L$ - the amplitude of stretching; $M$ - the piezo motor](image)

**Parameters**

The initial size of each film is $L_0 = 6$ mm while the amplitude of stretching or compressing is $\Delta L = 5$ mm, so the total length of the stretched film is $L = L_0 + \Delta L$. The piezo motor $M$ moves at constant velocity $U = [0.5 \ 2 \ 5 \ 10]$ mm/sec. A characteristic velocity of the problem is the velocity of the Frankel’s film extraction $U \sim 2$ mm/sec observed in Chapter 3 (Figure 3.24) after motor stops. Our velocity range was chosen around this reference value.

The chemical solutions are $S_1$ and $S_2$ with infusion flux $Q = 0.05$ ml/min. The hyperspectral camera $C_2$ records at 25 fps.

The physical parameters of the system are the extracted and the initial film thicknesses $h(t); d_1(t)$ - the size of the thin film (the initial and then the stretched).
5.4.2 Thin film volume per unit length

The hyperspectral camera $C_2$ gives us spectral information for all the wavelengths along one chosen line on the film. For this set of experiments we use the software of the hyperspectral camera, which provides us a spatio-temporal image during measurements (see Figure 5.11), each pixel of this image represents a continuous curve of incoming light intensity versus wavelength, so we have an image consisting 3D information $(z, t, I(\lambda))$. The free meniscus is at $z_0$, the moving frame is at $z_m$. Provided colors of the film in Figure 5.11 depend on the calibration of the hyperspectral camera and do not correspond exactly to the colors of the film but we still can get ideas of Frankel’s film extraction qualitatively from this image.

As each image has 3D information (space-time-spectrum), uploading it into Matlab and applying the formula 2.1 we can build a film thickness map $h(z, t)$.

In order to calculate total volume per unit length (let’s call it "volume" to simplify in the following explanations) for the film, we have to sum all the thicknesses multiplied by pixel size along one chosen line, which corresponds to the chosen time moment.

$$V(t) = \int_0^{L(t)} h(z, t) dz$$  \hspace{1cm} (5.1)
We define $L(t)$ like the distance between the meniscus $z_0$ and the solid frame $z_m$. At time $t_0$:

$$V_0(t_0) = \int_0^{L(t_0)} h(z, t_0)dz = \int_0^{L_0} h(z, t_0)dz$$  \hfill (5.2)

Let’s call this value - the initial volume $V_0$. After motor motion at $t > t_0$ we have an extraction of a thicker film from the meniscus at $z_0$ and we define the boundary between the extracted and the preexisted films as $z_f(t)$ (see notations in Figure 5.11). The distance taken by the initial volume $V_0$ between the moving frame at $z_m$ and the frontier $z_f(t)$ has changed. To express its value we write the total volume as:

$$V(t) = \int_{z_0}^{z_f(t)} h(z, t)dz + \int_{z_f(t)}^{L(t)} h(z, t)dz,$$  \hfill (5.3)

where

$$V_0 = \int_{z_f(t)}^{L(t)} h(z, t)dz.$$  \hfill (5.4)

This relationship defines the distance $d_1(t)$ which is taken by $V_0$, i.e. by the material system present in the film at initial time by the relation

$$d_1(t) = L(t) - z_f(t).$$  \hfill (5.5)

For all the experimental data we want to compare $d_1(t)$ obtained from different configurations of stretching/compressing. The Frankel’s film was extracted from the meniscus at $z_m$ either, however, because of the instability discussed in Chapter 4, a stratification occurs and the initial thin film is in the top part of the film, between $z_m$ and $z_f(t)$.

### 5.4.3 Configurations

For the following experiments we use several configurations of thin film stretching or compressing. The first one is the configuration, when only film $F_3$ is stretched (see Figure 5.10, let’s call it (0) configuration in the following). Then, looking at Figure 5.12 we have four others configurations:

1. $F_3$ and $F_4$ are stretched while $F_1$ and $F_2$ are at rest;
2. $F_3$ is stretched, $F_4$ is compressed, $F_1$ and $F_2$ are at rest;
3. $F_3$ is stretched, $F_1$ and $F_2$ are compressed, $F_4$ is at rest;
4. $F_3$ is stretched, $F_1$ and $F_2$ are stretched too, $F_4$ is at rest;

Figure 5.12: (1) - $F_3$ and $F_4$ are stretched, $F_1$ and $F_2$ are at rest; (2) - left part of the system is at rest, $F_3$ is stretched, $F_4$ is compressed; (3) - $F_3$ is stretched, $F_1$ and $F_2$ are compressed, $F_4$ is at rest; (4) - $F_3$ is stretched, $F_1$ and $F_2$ are stretched too, $F_4$ is at rest

These configurations were chosen to see the influence of the neighbouring films on the behavior of the observed one. In the first (1) and the second (2) configurations, we try to notice the influence of the neighbour which is at the same side, on the third (3) and the fourth (4) - neighbours from another side of the system. Under deformation, the soap film might behave as a monolayer between $F_1$ and $F_3$ or as a monolayer between $F_2$ and $F_4$. Is there a liquid transfer through the menisci or through the horizontal film $F_5$ which we do not deform?
5.4. DYNAMIC CONDITIONS. NEIGHBOURING FILM INFLUENCE

Or it is just stretching of $F_3$? These questions we want to answer in the following.

5.4.4 Experimental protocol

1. Choosing one of five configurations for the piezo motors connected as a daisy chain.

2. Choosing a chemical solution ($S_1$ or $S_2$) and adjusting the infusion flux $Q=0.05$ ml/min.

3. Immersing all the system into a reservoir with a chemical solution, waiting for 5 minutes for the soap film to have a steady thickness.

4. Running the hyperspectral camera, we choose $t_{all}=100$ sec (time from the beginning to the end of recording for the hyperspectral camera) for $U = 0.5$ mm/sec and $t_{all}=40$ sec for $U = 2, 5$ and $10$ mm/sec.

5. At time $t_0$ running the piezo motors, they stop at $t_s$ and we continue recording up to $t_{all}$.

6. Saving the data as 3D cube ($z, t, I(\lambda)$).

7. Repeating three times for each velocity value and for each chemical solution.

5.4.5 Results

In order to compare the data for $d_1$ as a function of $t$, for each configuration from (0) to (4) we plot $d_1(t)$ for one velocity value $U$ and as a result we have one figure $d_1(t)$ for five configurations.

In Figure 5.13 we have the dependence $d_1(t)$ for configurations (0) - (4) for $S_2$ chemical solution. Looking at these graphs we can say that the initial distance $d_1(t_0)$ has a value of around $d_1(t_0) = 6.5$ mm, which corresponds to the initial film length. Then it increases up to the maximum value. This corresponds to the stretching of the initial film, the film behaves like a spring, it stretches in an elastic regime. Afterward, this action produces the force which pulls the new extracted film out the meniscus, so the initial film takes less place in the stretched film and this is the decreasing part on each curve. The same behavior of soap film under stretching we discussed in more details in Part II (Chapter 3).

For all the curves in Figure 5.13 we see the very strong separation between pulling (red) and pushing (blue) configurations. Pulling a neighbours of the $F_3$ (see Figure 5.12, configurations (1) and (4)) induces a bigger extension of the initial film and smaller extracted film
compared to configurations (2) and (3). Pushing a neighbours of the observed $F_3$, we assume liquid transfer. More liquid we push, more liquid we transfer, this may explain a difference between red and blue lines in Figure 5.13.

The results for the same experimental series but another chemical solution $S_1$ are provided in Figure 5.14. We see the same tendency as for the previous case with dodecanol in the solution. For the pushing situations (2) and (3) the initial volume recovers faster its value.

Other features can be observed, which should require more experiments to be confirmed.

Comparing for configurations (0) - (4) with two different chemical solutions $S_1$ and $S_2$, we see that after reaching its maximum value, $d_1(t)$ decreases faster for the chemical solution $S_1$. The maximum position of $d_1(t)$ is bigger almost for all the velocities for the solution $S_1$. Less rigid film for the solution $S_1$ can be extended bigger than more rigid for $S_2$. The example for the configuration (0) is presented in Figure 5.15.

To observe the influence of the piezo motor velocity $U$ on $d_1(t)$, we plot for (0) configuration $d_1(t)$ for all tested velocities. The result is shown in Figure 5.16 (we stretch only $F_3$), the velocity $U$ plays a role only during the initial film extension. For all velocities the $d_1(t)$ increases during the piezo motor motion first and then it decreases due to the extraction of thicker film from the meniscus.
5.5 Conclusion

The key conclusion of the first experiments with the SET3 is that in the system of five connected films neighbouring films plays an important role in the behavior of the observed one. The provided experiments open a question for more detailed investigations in this geometry.

I started my PhD with the design of the SET3 and performing the first tests on it, we discovered that the extracted Frankel’s film, which is on the top of the initial one, is always unstable. This question required more discussion that is why I switched to another set up (SET1 and SET2). The gravitational 2D instability in a soap film potentially influence all
the measures of thickness or velocities made in the non-horizontal films, so this question
needed to be understood first. That is why the instability in a soap film under stretching
became one of the main topics of my thesis.

As the SET3 was tested and it has shown the results consistent with what we expected,
it was decided to improve the set up by making all the pieces from a metal. It allowed us to
avoid problems with pieces rigidity, which, in turn, improved the precision of measurements.
All the experiments of Adrien ([10]) were performed on the new "metallic set up".

The question of liquid transfer through the central film $F_5$ is still open and the model to
describe the dynamics of film extraction is needed to be developed. The film behaves like
elastic and viscous elements, similarly, as we discussed in Paragraph 3.4.4 by building the
model of Frankel's film extraction in a horizontal film.
5.5. CONCLUSION

Figure 5.14: $d_1(t)$; $S_1$ solution. Red line $- \ (1)$; red line with stars $- \ast \ (4)$; blue line $- \ (2)$; blue line with stars $- \ast \ (3)$; black line $- \ (0)$
Figure 5.15: Comparison $d_1(t)$ for $S_1$ and $S_2$ chemical solutions for the configuration (0)
Figure 5.16: Comparison different velocities $U$ for the configuration (0); $- U = 0.5 \text{ mm/sec}$; $- U = 2 \text{ mm/sec}$; $- U = 5 \text{ mm/sec}$; $- U = 10 \text{ mm/sec}$;
Chapter 6

Conclusion

In this thesis, we studied the question of a soap film under stretching and the Rayleigh-Taylor instability appearing, when after stretching a thicker extracted film is produced on the top of a thin one. We also observed the influence of neighbouring films on the stretched in a foam sample at millimetric scale.

In Chapter 1 we introduced the problem including a soap film under deformation, which leads to an extraction of a new thicker film quantitatively obeying Frankel’s law with the assumption of incompressible interfaces.

In Chapter 2 we explained the construction of the set up for creating three soap films connected through one common meniscus. The design and development of the set up were made by our team. The first version, which is used for all my experiments, is made of plastic but recently we improved it and built a new version from metal, which is more rigid and precise. All further experiments will be performed on a new set up. In this chapter we also explained the method of using the hyperspectral camera for film thickness measurements, which allows us to get a much wider understanding of thickness behavior of the entire film compared to the results from a spectrometer which is usually used for such kind of measurements.

In Chapter 3 we studied Frankel’s film extraction by stretching a soap film. We suggested the classical Maxwell’s model consisting of a linear spring element and two linear viscous dashpot elements to describe the problem. We characterized stress-strain relations in the model and built the expressions for the distance, taken by an extracted film with respect to a time \( d_2(t) \). Then, after performing an experimental series with different chemical solutions, we observed the same concept on the experiment. The initial film under stretching behaves first as an elastic string. New thicker film extraction corresponds to a viscous regime. The optical devices used in the experiments let us well control the boundary between thicker
and thinner parts of the film and built the same expression $d_2(t)$, which, in turn, might be compared to the one from the model. We found a very good correspondence between the model and the experimental results (both at a long time and in the transient zone). Based on this fact we obtained quantitatively the value of the Gibbs elasticity $E$ of the film. This Maxwell’s model should still be confronted to the experimental results of the literature, especially of the T1 process.

In the same chapter, we produced a thickness map $h(z, t)$ and we tried to compare the results with Frankel’s law. We found a difference in thickness in two times (predicted is bigger during extension and smaller afterward). During extension, we might not reach a steady regime, which can explain smaller thickness at the beginning and we had plenty of lost information while recording the spectrum by the hyperspectral camera, which might lead to the thickness difference afterward. As it was first time used and tested, the experimental protocol with the hyperspectral camera should be improved to obtain better spectral information.

In Chapter 4 we introduced and described an example of the 2D Rayleigh-Taylor instability, which is taken place in a stretched soap film when the thicker extracted film is found on the top of the thinner one. As it was not described previously, we developed and explained a linear model of the problem. In this thesis, I first considered the case of the unlimited size of extracted film $d_2$ and compared the values of the wavelength of the instability $\lambda$ and the growth rate $n$ with experimental ones. We carried out a big series of experiments trying to tune several experimental parameters as $\bar{g}$ the effective gravity and $U$ the velocity of stretching. We discovered that one important physical control parameter is the size of the thick film $d_2$. So we improved the model with a cut-off. With this correction, the model provides the right tendency but it remains an overestimation, which might be explained by the fact that the system is not steady (another smaller value of $d_2$ is needed to develop the instability) or the boundary condition at the top used in the model, (a vanishing velocity) is not the actual one.

The question about the non-linear regime of the instability requires new experimental and theoretical study. During the last months of my PhD I worked with the internship student on the experimental setup design allowing to follow the trajectory of a drop developed during instability and to measure its velocity. There are three forces acting on the drop - the viscous force, the weight of the drop and the buoyant force. Balancing them we can estimate the value of the surface viscosity. We tried to improve the set-up in order to use it as a rheometer for the interface shear viscosity.

In Chapter 5 we gave the first ideas about soap film coupling in a foam sample at milli-
metric scale. The biggest contribution to this question was the design and the first tests of the setup. We have got a clear understanding that there is an interface transfer across the menisci. The key boundary condition for local flows in foams is the transfer velocity and its first measurements were already performed [10] due to well tested and developed set up. We have also patented this invention as rheometer for liquid films.
Appendices
Appendix A
Rayleigh Taylor like instability in a foam film

Evgenia Shabalina, Antoine Bérut, Mathilde Cavelier, Arnaud Saint-Jalmes and Isabelle Cantat

Univ Rennes, CNRS, IPR (Institut de Physique de Rennes) - UMR 6251, F- 35000 Rennes
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It is well known, since the seminal work of Mysels et al., that the thinner parts of a foam film go up by gravity, whereas the thicker parts go down. Preparing a foam film in a controlled way, so that the top part of the film is much thicker than the bottom part at initial time, we show that this situation is indeed unstable under gravity. The observed instability is identified as a Rayleigh-Taylor like instability and studied in the linear regime. The wavelength and the growth rate are measured as a function of the effective gravity, and as a function of the thick film extension. We theoretically show that all wavelengths are unstable, as the surface tension stabilizing the small wavelengths in the classical bulk Rayleigh Taylor instability has no significant equivalent in a foam film. The fastest mode is analytically determined and is in qualitative agreement with the experimental observations.

I. INTRODUCTION

Inside an aqueous foam, gas bubbles are packed and separated by thin liquid films. The existence of such a foam directly relies on the ability to stabilize these films at a finite thickness. For that purpose, surfactants needs to be added to the aqueous phase: they can then adsorb on the gas-liquid interfaces, and eventually provide some stabilizing mechanisms.

In parallel, both when a foam ages and when it flows, the arrangement of the bubbles does not remain still, and bubbles keep swapping their relative positions. This implies that the liquid films are contracted, elongated, deformed and that some films vanish while others are created. As a consequence, some liquid flows are induced inside the films, and the liquid is dynamically distributed between the films and the menisci to which they are connected (also known as 'Plateau borders').

The liquid flow in such surfactant-stabilized films has some specific features. On one hand, the liquid is strongly confined between the two interfaces covered by surfactants, and any flow relatively to these interfaces is highly dissipative; this effect appears to be a key element for understanding the high effective viscosity of liquid foams [1, 2]. On the other hand, the in-plane mobility of the liquid is very high, so that turbulent flows can be easily induced in dedicated experimental configurations. In consequence, surfactant-stabilized liquid films have been used as a model system to reach high Reynolds numbers and to study turbulence in 2D [3, 4].

For all these reasons, understanding all the equilibrium and dynamical properties of an aqueous foams requires to unravel the interactions and flows acting at the scale of the liquid films. Many experimental results have been collected by monitoring a single liquid film. A large literature is available on horizontal films held on solid frames, especially in the framework of the 'thin film balance' apparatus [5, 6]. With this setup, one can focus on the small thickness regime (due to high capillary suction by the meniscus), and on the repulsive and attractive forces between the two surfactant-covered layers. Over the years, such experiments have brought many new insights on the required conditions for film stability, as well as on confinement effects (like stratification, [5, 6]). However, gravity plays no role in these horizontal films, and only smooth radial flows - due to capillary suction towards the surrounding meniscus - can be monitored. Thus, this setup is not suited for investigating flows induced during fast dynamical changes of film area, like those occurring during bubble swapping.

On the opposite, other experiments have been performed on single vertical films, most generally held on large (centimeters) solid frames. In such conditions, the gravitational drainage of a film can be studied [7–9, 11, 12]. Subtle effects, like 'pinching' or 'marginal regeneration' [7, 13], have then been evidenced, illustrating the non-trivial flows occurring in such vertical films. It is indeed well known since the seminal work of Mysels [7] that thinner parts of a foam film move up by gravity, whereas the thicker parts go down. A continuous injection of surfactant solution inside a vertical film can also be done to sustain high film thickness of a few microns. As pointed out previously, 2D turbulence has actually been studied with such sustained vertical films [3, 4]. Nevertheless, these films are then far from those found in usual foams, both in terms of size, thickness and velocity scales.

More recently, to fill the gap between a single isolated film and a 3D opaque foam, experiments have been performed on system which can be considered as the elementary building blocks of a foam, meaning a few films connected to one (or a few) meniscus [10, 14–20]. The main goal is then to identify non-trivial features occurring at the connections between free films and menisci. Together with the fact that these clusters of films are relatively easy to build, one of their advantages is that they allow to visualize and to monitor the various flows, deformations and out-of-equilibrium film thicknesses [21]. Another advantage is that these elementary clusters can be activated on purpose: one can trigger re-organization of films, control flows in films, etc... Also - as shown in the following article - one can also easily change the setup orientation to tune the effects of gravity. In parallel, by continuing to study such clusters of films and menisci, one can also expect to shed light on the efficiency of these elementary building blocks to mimic 3D
foams. In fact, how far one can transpose the observations made on film clusters to any foams remains to be fully elucidated, especially in terms of bubble sizes, liquid fractions, or chemical formulation.

Here, we present results performed on a new type of film/meniscus cluster, and allowing us to prepare one of the foam film in a controlled way, so that the top part of this film is initially much thicker than the bottom part. We show that this situation is indeed unstable under gravity and we identify this instability as a Rayleigh Taylor instability [22]. The wavelength and the growth rate are measured in the linear regime as a function of the effective gravity, and as a function of the film extension. We theoretically show that all wavelengths are unstable, as the surface tension stabilizing the small wave lengths.

The two other films \( F' \) and \( F'' \) are of area \( w \times L_{\text{lat}} \), with \( L_{\text{lat}} = 13 \text{mm} \).

We define as \( x \) the meniscus direction, and as \( (x, z) \) the plane of the film of interest \( F \). We use two variants of the same set-up. The first one (set-up A) is on a table which inclination \( \theta \) can be controlled in the range \([0 - 9^\circ]\), so that the direction \( z \) makes an angle \( \theta \) with the horizontal. With these conventions, the projection of the gravity vector on the film plane is \( -\vec{g}e_x \), with \( \vec{g} = g \sin \theta \). In the second one (set-up B), the film \( F \) is placed vertically, with the free meniscus at the top, so \( \theta = \pi/2 \). We describe below the protocol for set-up (A), the one for set-up (B) being similar.

We used two different foaming solutions. Solution \( S_a \) is made of Sodium dodecyl sulfate (SDS) at 5.6 g/L and glycerol at 15 % of volume. In solution \( S_b \) we added dodecanol at 0.05 g/L. The bulk viscosity is \( \eta = 1.5\times10^{-3} \text{ Pa.s} \), the density is \( \rho = 1.05 \text{ kg/m}^3 \) and the surface tension is \( \gamma = 29 \text{ mN/m} \) for the \( S_a \) and 30mN/m for \( S_b \) [23]. The films are prepared by immersing the frame into a vessel containing the foaming solution and by removing slowly the vessel. The film drains during 15 s, then is extended at \( t = 0 \), by moving the piezo stage at velocity \( U_m = 80 \text{ mm/s} \), so that \( L(t) \) varies from \( L_0 = 6 \text{mm} \) to \( L_0 + \Delta L = 19 \text{ mm} \). The film \( F \) is observed at a frame rate of 60 Hz with a color camera Imaging Source DFK 23 UM021, with 1280 \( \times \) 960 pixels. The incident light is in the \((x, y)\) plane and the camera is at the mirror position.

An example of film evolution during and after the displacement of its moving edge is shown in Fig. 2, for the case of set-up (B). The images are all cropped just below the free meniscus (see the few black lines at the top of the images). This meniscus remains almost at a constant position during the deformation. The moving edge appears in black at the bottom of the film. It begins to move at image (a) and stops at image (d).

The shape of the film \( F \) remains flat and rectangular and its area \( wL(t) \) increases at the rate imposed by the linear stage motion. As already shown in [17], this area variation is first insured by an extension of the foam film initially present (denoted as the initial film in the following), which lead to the increase of the surface tension. As the film tension becomes larger than the meniscus tension, the initial film becomes able to pull on the meniscus and to extract volume and interface from this reservoir: new pieces of film, hereafter called the Frankel’s films, are extracted from the menisci bounding the deformed film [7]. Consistently, the initial film is first stretched and get thinner between images (a) and (d). Colors indicate for

II. EXPERIMENTAL SET-UP

The set-up and the reference frame are schematized in Fig. 1. A horizontal free meniscus of width \( w = 5 \text{ cm} \) is connected to three flat rectangular foam films. For one of these films, denoted by \( F \) in the following, the edge parallel to the free meniscus can translate along the lateral edges, and its position is controlled by a piezo translation stage. Its area is thus \( w \times L(t) \) with \( L(t) \) the distance between the moving edge and the free meniscus.

The wavelength and the growth rate are measured in the linear regime as a function of the effective gravity, and as a function of the film extension. We theoretically show that all wavelengths are unstable, as the surface tension stabilizing the small wave lengths in the classical bulk Rayleigh Taylor instability has no significant equivalent in a foam film. The fastest mode is analytically determined and is in qualitative agreement with the experimental observations.

A. Qualitative description of the flow

An example of film evolution during and after the displacement of its moving edge is shown in Fig. 2, for the case of set-up (B). The images are all cropped just below the free meniscus (see the few black lines at the top of the images). This meniscus remains almost at a constant position during the deformation. The moving edge appears in black at the bottom of the film. It begins to move at image (a) and stops at image (d).

The shape of the film \( F \) remains flat and rectangular and its area \( wL(t) \) increases at the rate imposed by the linear stage motion. As already shown in [17], this area variation is first insured by an extension of the foam film initially present (denoted as the initial film in the following), which lead to the increase of the surface tension. As the film tension becomes larger than the meniscus tension, the initial film becomes able to pull on the meniscus and to extract volume and interface from this reservoir: new pieces of film, hereafter called the Frankel’s films, are extracted from the menisci bounding the deformed film [7]. Consistently, the initial film is first stretched and get thinner between images (a) and (d). Colors indicate for
example a thickness of 450 nm in (b) (dark blue), and 350 nm in (d) (yellow). In image (c) the gray bands appearing at the top and bottom are the Frankel’s films extracted from the menisci. The absence of interference colors indicates a thickness larger than 1 µm. They are thus much thicker than the initial film and a sharp frontier separates them from it.

The Frankel’s film extracted from the bottom meniscus (located on the moving edge) remains stable, whereas the one extracted from the top meniscus (the free one) destabilizes after a fraction of second. Its frontier with the initial film exhibits a relatively regular and sinusoidal shape for a while (image (d)) and then produces drop-like structures reminiscent of a usual Rayleigh Taylor instability (images (e) and (f)). These drops of thick film eventually detach from the top Frankel’s film and fall through the initial film until they merge with the bottom Frankel’s film (images (g) and (h)).

In this paper, we focus on the linear regime of the gravitational destabilization of the top Frankel’s film.

B. Image processing

To quantify the destabilization process, we analyzed the time evolution of the frontier between the top Frankel’s film and the initial film. This frontier is automatically detected using a matlab routine (see Fig. 3). The extrema of the frontier are then identified, and the experimental wavelength $\lambda^{exp}$ is defined as the average distance (along x) between minima, measured in the first image where oscillation are visible (image $I_1$). The height $d_2$ of the top Frankel’s film is measured on $I_0$, the last image before $I_1$.

Few minima are followed between $I_1$ and the following image $I_2$ to determine the growth rate $n$. We assume that the position along z of a minimum obeys $z(t) = a \exp(at)$, with $a$ the initial noise ($a < 0$ in this case) and $n$ the growth rate. We thus define the experimental value of the growth rate as

$$n^{exp} = \frac{1}{t_2 - t_1} \ln \left( \frac{z_m(t_2)}{z_m(t_1)} \right).$$

Finally the thicknesses of both the initial film and the Frankel’s film are deduced from the data obtained with the spectral camera. This camera makes the image of a line \(\mathcal{L}\) perpendicular to the free meniscus and measures the spectrum of the light reflected by each point of \(\mathcal{L}\): the intensity $I$ of the light of wavelength $\lambda$, in the range $[375 - 1010]$, reflected by the piece of film at the position $z_i$ and of size $dz \ dx$ is recorded by the pixel $(i, j)$ of the 2D sensor. The distance $dx$ is fixed by the width of a slit in the camera and $dz = 54 \, \mu m$ is the spatial resolution. The light trajectory is in the plane $(y, z)$ and the incidence angle on the film is $\phi = 45^\circ$. The lamp is a usual halogen lamp. A typical image obtained after motor motion for the horizontal case is shown in Fig. 4.

From such images, we get $h(z, t)$ using the relation

$$I(\lambda) \propto 1 - \cos \left( \frac{4\pi hn}{\lambda} \left( 1 - \frac{\sin^2 \phi}{n^2} \right)^{1/2} \right),$$

with $n$ the optical index of the solution.

The transition between the initial film and the Frankel’s film occurs over a distance $\delta$ much smaller than the typical film size, and both parts of the film are thus well defined. We denote as $h_1$ and $h_2$ the average thicknesses respectively of the initial and top Frankel’s films, computed on the last image before the time of destabilization.
III. EXPERIMENTAL RESULTS

A. Linear Regime: wavelength and growth rate

The first series of data have been obtained with the solution $S_a$, on the inclined table (set-up A). In this series the film thicknesses and initial sizes are kept as constant as possible and the effective gravity is varied. At the onset of the instability, we have $h_1 = 0.4 \pm 0.1 \mu m$, $h_2 = 1.2 \pm 0.2 \mu m$, $d_1 = 8$ mm and $d_2 = 4.2 \pm 0.7$ mm. The observed wavelength is shown in Fig. 5 as a function of the effective gravity $g$: the wavelength slightly decreases when the angle increases. The data are rather dispersed, and we checked that this variability was not correlated with the fluctuations of the system characteristics: in Fig. 5, we shows that $d_2$, given by the color chart, is not correlated to the wavelength, at a given angle. Similarly the film thickness variations cannot explain the data dispersion.

In Fig. 5, we show that the growth rate increases with the effective gravity. Finally, as expected, the frontier between the thin and thick films is stable when the film is perfectly horizontal ($\theta = 0$).

Very similar results are obtained with the solution $S_b$ (with dodecanol) in almost the same conditions, as shown in Fig. 6. In that second series, $h_1 = 0.5 \pm 0.1 \mu m$, $h_2 = 3 \pm 0.5 \mu m$ and $d_2 = 3 \pm 0.3$ mm.

The vertical set-up (set-up B) has been used with solution $S_b$. In that case, the effective gravity is fixed, and we modify the characteristics of the thin and thick films by changing the motor velocity from $U_m = 5$ mm/s to the maximal rate $U_m = 112$ mm/s. As the stretching of the initial film and the extraction of the Frankel's film are two competing effects occurring simultaneously during and after linear stage motion, the various geometrical parameters $h_1$, $h_2$, $d_1$ and $d_2$ can not be varied independently. This series has been obtained without the spectral camera and the thicknesses are deduced from the observed colors, compared to a color chart.

The obtained parameter range is $[0.2 - 1.2]$ mm for $d_2$ and $[0.3 - 0.7] \mu m$ for $h_1$. We only have an estimation for $h_2$, which is in the range $1 - 2 \mu m$. The thin film height is always much larger than $d_2$ and plays no role. Fig. 7 shows the wavelength and the growth rate as a function of the thick film height $d_2$. The color indicates the thin film thickness $h_1$, with an error bar of 100 nm.

These experimental data are discussed in section V, and compared to the model developed below.
with the prefactor 4.5 (instead of the expected 19.5) and by eq. 11 with the prefactor 1/2.8). The color indicates the value of the thick film height $d_2$, in mm.

IV. THEORY: LINEAR STABILITY ANALYSIS

A. Model assumptions

The Frankel’s film extracted at the bottom is stable. It plays no role in the instability and will not be taken into account in the model, which will focus on the coupling between the thick film at the top, of thickness $h_2 \approx 2\mu m$ and height $d_2$ (region 2 of the film) and the thin film at the bottom, of thickness $h_1 \approx 0.5\mu m$ and height $d_1$ (region 1). The fluctuations of $h_1$ or $h_2$ with time and space are much smaller than the thickness difference $h_2 - h_1$, so a uniform thickness will be assumed in each region. The transition between one region to the other occurs on a typical length $\delta$ of the order of 100$\mu m$ (see Fig. 4 and [17]). This length is smaller than the observed wavelength and a sharp transition is therefore assumed between both regions. The frontier is located along the line $z = d(x, t)$ which will be predicted by the model.

The fluid dynamics is governed by the lubrication equations and the velocity and pressure fields $\mathbf{V}$ and $P$ thus obey, in each part of the film,

$$ V_i = \frac{1}{\eta} \left( \frac{y - h_1}{2} \right)^2 (\nabla P + \rho g) + v_i , \quad (3) $$

with $v_i$ the interfacial velocity in the domain $i$. The pressure gradients $\nabla P$ are governed by the Laplace pressure and are localized at the frontier between both films, where the interface curvature is non zero. They scale as $\gamma(h_2 - h_1)/\delta^3 \sim 10^4 \text{m}^2/\text{s}$, and are thus comparable to the gravity term $\rho g$ in the frontier vicinity. The induced Poiseuille flow scales as $(\rho g + \nabla P)h^2/\eta \sim 10\mu m/s$, whereas the observed velocities are of the order of 10 mm/s: the velocities are thus dominated by the interfacial velocities $v_i$. In the following, we assume that the velocity is uniform across the film and is equal to $v_i(x, z, t)$, and that the pressure is homogeneous and equal to the air pressure. One important consequence is that an elementary piece of film of volume $h_i dx dz$, taken either in the thin or in the thick film, is a closed material system. Especially, there is no liquid flux across the frontier $d$ between both film regions.

We assume that the instability can be described using an inextensible interface model. This assumption implies
that each piece of film $dx dz$ keeps a constant thickness with time.

Finally, the instability begins to grow during the Frankef's film extraction, so the reference state, characterized by $d_1$ and $d_2$, depends on time. However, in the following, we make the crude assumption that the instability time scale is much smaller than the extraction time scale and that $d_1$ and $d_2$ can be assumed to be constant. To summarize, the main physical processes can be reproduced assuming that the parts of film (1) and (2) are of constant and uniform thickness $h_1$ and $h_2$, and of constant area $wd_1$ and $wd_2$.

The evolution of the frontier is governed by the gravity $\bar{g}$, the surface tension of the film $\sigma$ (taking into account both interfaces) and the shear viscosity of the film $\mu_s i = \eta h_i$. The interface viscosity is typically of the order of $\nu_s \approx 10^{-6}\text{kg s}^{-1}$ [23], much larger than $\eta h_i \sim 10^{-9}\text{kg s}^{-1}$. The film shear viscosity can thus be approximated by $\mu_s = 2\eta_s$ in both parts of the film.

**B. Reference state**

In the reference state, the frontier $d_0(x)$ between parts (1) and (2) of the film is a horizontal straight line and the reference for $z$ is chosen so that $d_0(x) = 0$. The interface velocities are vanishing and the whole system is at rest. The force balance in the $z$ direction on a piece of film $dx dz$ is therefore simply

$$0 = \frac{\partial \sigma}{\partial z} - \rho \bar{g} \bar{h}$$ (4)

We thus get

$$\sigma_0 = \rho \bar{g} h_1 z + 2\gamma_0$$ and $$\sigma_2 = \rho \bar{g} h_2 z + 2\gamma_0$$ (5)

in the reference state, with $\gamma_0$ the interface tension reference, taken arbitrary at $z = 0$. It is analogous to a hydrostatic pressure in a 3D problem.

**C. Scaling laws**

In order to determine the stability of the reference state, a frontier shape $d(x, t) = \varepsilon e^{nt} e^{kx}$ is assumed. Before dealing with the full linear stability analysis we first derive scaling laws for $n(k)$ based on simple force balances.

We first deal with the system (1) shown in Fig. 8(left), made of piece of film of size $\lambda \times \lambda$ across the frontier, with $\lambda = 2\pi/k$ the perturbation wavelength.

As the system size in the $z$ direction is of the order of the wavelength, all physical quantities remain invariant in the $x$ direction along the top and bottom boundaries. The surface tension is thus independent of $x$ along these lines and denoted by $\sigma + \Delta \sigma$ at the top boundary and $\sigma$ at the bottom.

The mass of the system is (disregarding any numerical prefactor) $m = \lambda^2 \rho (h_1 + h_2)$. The acceleration is vanishing by symmetry: the left part moves down and the right part moves up. Similarly, the viscous forces on both sides compensate each other. Finally, the resulting force due to the surface tension is $\lambda \Delta \sigma e_z$ and the weight is $-\lambda^2 \rho \bar{g} (h_1 + h_2) e_z$. The force balance on the system thus imposes

$$\Delta \sigma \sim \lambda \rho \bar{g} (h_1 + h_2).$$ (6)

The equilibrium relation 4 thus remains true in the out of equilibrium case, far enough from the frontier.

We now focus on the subsystem (2), made of the right part of the previous system, moving upwards (see Fig. 8(right)). The surface tension contribution is still $F_{\sigma} = \lambda \Delta \sigma$, but it now slightly overcompensates the gravity $F_g = -\lambda \rho \bar{g} [h_1 (\lambda + \varepsilon) + h_2 (\lambda - \varepsilon)]$. The resulting force is $F_{\sigma} + F_g = \lambda \rho \bar{g} e_z(h_2 - h_1)$.

One damping term is the film inertia, scaling as $\lambda^2 \rho (h_1 + h_2) \varepsilon^2$. Another arises from the interfacial viscous forces: on both lateral sides of the system, the velocity gradients scale as $n \varepsilon / \lambda$ (see Fig. 8(right)) and the resulting force thus scales as $-\mu_s n \varepsilon$.

Finally, we can define a line tension $T$ between the domains (1) and (2). The order of magnitude of this tension can be determined from the area excess induced by the frontier. As the thickness varies from $h_1$ to $h_2$ over a distance $\delta$ perpendicular to a local line element $dl$, the area of this transition region is $a_{nl} = dl (\delta^2 + (h_2 - h_1)^2)^{1/2} \sim dl \delta(1 + (h_2 - h_1)^2/(2\delta^2))$ (see Fig. 9). The excess area due to the thickness variation is thus $(h_2 - h_1)^2/(2\delta)$ per unit length. The resulting line tension $T$ scales as $\sigma (h_2 - h_1)^2/\delta \sim 10^{-10}\text{N}$, and the associated force on the system scales as $-\varepsilon \sigma (h_2 - h_1)^2/(\lambda \delta)$ (see Fig. 8(right)).
Finally, the driving force for the instability is the excess of gravity, scaling as \( \lambda^1 \), and the three damping and stabilizing terms are the line tension, the interfacial viscous forces and the inertia, scaling respectively as \( \lambda^{-1} \), \( \lambda^0 \) and \( \lambda^2 \).

The line tension term dominates at very small wavelength. Balancing the gravity excess and the line tension, we show that it ensures the stabilization of perturbations having a wavelength smaller than \( \lambda_c \), with

\[
\lambda_c = \left( \frac{\sigma(h_2 - h_1)}{\rho \delta} \right)^{1/2} \sim 10 \mu m . \tag{7}
\]

All length scales smaller than this cut-off are stable. However, as \( \lambda_c \ll \delta \), the model of sharp interface is not valid anymore at this scale, and the line tension will be neglected in the following.

Just above \( \lambda_c \), the instability is damped by the viscosity and the growth rate is obtained by balancing gravity and interfacial viscosity terms

\[
\eta_{visq} \sim \frac{(h_2 - h_1) \lambda \overline{\rho \delta}}{\mu_s} . \tag{8}
\]

Similarly, in the large \( \lambda \) limit, the damping term is the inertia and the growth rate scales as

\[
\eta_{inert} = \left( \frac{(h_2 - h_1) g}{(h_1 + h_2) \lambda} \right)^{1/2} . \tag{9}
\]

The full numerical prediction, based on the approximations discussed above and on the equations established in the next paragraph, is shown in Fig. 10. The power laws given by eq. 8 and 9 are recovered at small and large \( \lambda \), respectively. There is a direct parallel with the usual Rayleigh - Taylor instability in 3D systems, surface tension playing the role of the pressure and film thickness replacing the density.

The fastest growth rate \( \eta_{th,1} \) is obtained for the wave length \( \lambda_{th,1} \) verifying \( \eta_{visq} \sim \eta_{inert} \). Using the prefactors obtained by comparison with the numerical prediction (see Fig. 11), we get

\[
\lambda_{th,1} \simeq 19.5 \left( \frac{\mu_s^2}{(h_2 - h_1)^2 \rho^2 \overline{\rho \delta}} \right)^{1/3} , \tag{10}
\]

\[
\eta_{th,1} \simeq \frac{1}{2.8} \left( \frac{(h_2 - h_1)^2 \overline{\rho \delta}}{(h_2 + h_1) \mu_s} \right)^{1/3} . \tag{11}
\]

For \( \mu_s = 10^{-7} \text{kg/s}^{-1} \), \( h_2 = 2 \mu m \), \( h_1 = 0.5 \mu m \) we find \( \lambda_{th,1} \) of the order of few millimeters, which may be comparable to the thick film height \( d_2 \). We thus expect a cut-off at \( d_2 \) of the scaling for \( \lambda \): wavelengths larger than \( d_2 \) can not grow, so if \( \lambda_{th,1} > d_2 \) then the fastest wavelength scales as \( d_2 \) and its growth rate is controlled by eq. 8 (see Fig. 10). This imposes, for the cases \( \lambda_{th,1} > d_2 \),

\[
\lambda_{th,2} \simeq 3.8 d_2 , \tag{12}
\]

and

\[
\eta_{th,2} \simeq \frac{1}{10.5} \frac{(h_2 - h_1) d_2 \rho \overline{\rho \delta}}{\mu_s} . \tag{13}
\]

Here again, the prefactors result from fits of the numerical solutions (see Fig. 11).

D. Equations in the bulk

These scaling laws clearly identify the physical processes, which are modeled below. The stability analysis of the usual Rayleigh-Taylor instability have been done for 2D systems in [24]. We re-establish the dispersion relation here, in the context of soap films, and extend it to finite size systems.

We define the interface velocity \( \mathbf{v}_i = (u_i, w_i) \) and the film tension \( \sigma = \sigma^0 + \delta \sigma \), with \( \sigma^0 \) given by eq. 5.
FIG. 11. Theoretical fastest wavelength (top) and associated time scale (bottom) as a function of the upper film size $d_2$. The parameters are $\lambda_1 = 0.5\mu m$, $\lambda_2 = 2\mu m$, $\mu_s = 10^{-7}kg/s^2$, and an infinite value of $d_1$. The angle $\theta$, governing the apparent gravity, verifies $\sin \theta = 0.01$, 0.1, 1 respectively for the red, magenta and blue lines. (Top) Horizontal dashed lines: 19.5$\lambda_{th,1}$ (eq. 10); black dashed line: 3.8$\lambda_{th,2}$ (eq. 12). (Bottom) Dashed lines: 2.8$\lambda_{th,1}$ (eq. 11); dot-dashed lines: 10.5$\lambda_{th,2}$ (eq. 13). Solid lines are the numerical solutions $\lambda_{num}$ and $\tau_{num}$ of the full system (eqs. 25-27, 30 and 35-36). The prefactors of the scaling laws eqs. 10-13 are fitted on these numerical solutions.

The assumption of inextensible interfaces leads to

$$\partial_x u + \partial_z w = 0$$

(14)

The force balance written on a film piece $dxdz$ of thickness $h$ (of value either $h_1$ or $h_2$) is, projected in the $x$ and $z$ directions

$$\begin{align*}
\rho h (\partial_t u + u \partial_x u + v \partial_y u + w \partial_z u) &= \partial_x \sigma + \mu_s (\partial_x u + \partial_z w) , \\
\rho h (\partial_t w + u \partial_x w + v \partial_y w + w \partial_z w) &= \partial_z \sigma + \mu_s (\partial_x w + \partial_z w) - \rho g h .
\end{align*}$$

(15)

(16)

At leading order we recover the equilibrium equation for the reference state eq. 4 and, at order 1 in $\varepsilon$,

$$\begin{align*}
\rho h u &= (ik \delta \sigma + \mu_s (-k^2 u + \partial_z u)) , \\
\rho h w &= \partial_z \delta \sigma + \mu_s (-k^2 w + \partial_z w) , \\
\rho h v &= -\partial_z w .
\end{align*}$$

(17)

(18)

(19)

Substituting $\delta \sigma$ and $u$ in eqs 17-19, we get an equation on $w$ only:

$$\mu_s \partial_z w - (\rho h n + 2k^2 \mu_s) \partial_z w + (\mu_s k^4 + \rho h n k^2) w = 0 ,$$

(20)

which solutions are, in domains $i = 1$ or 2,

$$w_i = A_i e^{kz} + B_i e^{-kz} + C_i e^{qz} + D_i e^{-qz} .$$

(21)

with

$$q_i = \left(\frac{\rho h n + k^2}{\mu_s} \right)^{1/2} .$$

(22)

and $A_i$, $B_i$, $C_i$ and $D_i$ coefficients we need to determine from the boundary conditions.

The $x$-component of the velocity and the dynamical surface tension in each domain can be deduced from $w$ as

$$\begin{align*}
\delta \sigma &= \frac{\rho h n}{k^2} \partial_z w + \frac{\mu_s}{k^2} (k^2 - \partial_z z) \partial_z w , \\
u &= \frac{i}{k} \partial_z w .
\end{align*}$$

(23)

(24)

E. Boundary condition at the interface

The normal velocity continuity at the frontier $w_1(d) = w_2(d)$ leads to:

$$A_1 + B_1 + C_1 + D_1 = A_2 + B_2 + C_2 + D_2 ,$$

(25)

and the tangential velocity continuity $u_1(d) = u_2(d)$ provides, using eq. (24):

$$k A_1 - k B_1 + q_1 C_1 - q_1 D_1 = k A_2 - k B_2 + q_2 C_2 - q_2 D_2 .$$

(26)

The continuity of the tangential stress

$$\mu_s (\partial_z w_1 + \partial_z w_1) = \mu_s (\partial_z w_2 + \partial_z w_2)$$

is

$$2k^2 (A_1 + B_1) + (q_1^2 + k^2)(C_1 + D_1) = 2k^2 (A_2 + B_2) + (q_2^2 + k^2)(C_2 + D_2) .$$

(27)

Finally, disregarding the role of the line tension, the continuity of the normal stress imposes

$$2\mu_s \partial_z w_1 + \partial_z \sigma \varepsilon + \delta \sigma_1 = 2\mu_s \partial_z w_2 + \partial_z \sigma \varepsilon + \delta \sigma_2 .$$

(28)

This expression involves $\varepsilon$ because $\partial_z \sigma$ is of order 0. Its value is obtained from the kinematic condition $\partial_t d = w_1 = w_2$:

$$n \varepsilon = w_1 .$$

(29)
The value of $\delta \sigma$ is given by eq. 23. After some reorganization proposed in [24] and given in Appendix 1, the condition becomes

$$0 = A_1 \left( \frac{R}{2} - \alpha_1 \right) + B_1 \left( \frac{R}{2} + \alpha_1 \right) + C_1 \frac{R}{2} + D_1 \frac{R}{2} + A_2 \left( \frac{R}{2} + \alpha_2 \right) + B_2 \left( \frac{R}{2} - \alpha_2 \right) + C_2 \frac{R}{2} + D_2 \frac{R}{2},$$

(30)

with

$$R = \frac{\bar{g} k h_2 - h_1}{n^2 h_1 + h_2},$$

(31)

$$\alpha_i = \frac{h_i}{h_1 + h_2}.$$  

(32)

**F. Limit of large $d_1$ and $d_2$**

The velocity remains finite so $A_2 = 0$, $B_1 = 0$, $C_2 = 0$, $D_1 = 0$. The system is then

$$A_1 + C_1 - B_2 - D_2 = 0,$$

$$k A_1 + q_1 C_1 + k B_2 + q_2 D_2 = 0,$$

(33)

$$2k^2 A_1 + (q_1^2 + k^2) C_1 - 2k^2 B_2 - (q_2^2 + k^2) D_2 = 0,$$

$$A_1 (R - 2 \alpha_1) + C_1 R + B_2 (R - 2 \alpha_2) + D_2 R = 0,$$

in agreement with the system eq. (111) in [24], for a problem in bulk, without surface tension.

This system has non trivial solutions only if its determinant is zero, which imposes, as established in Appendix 2,

$$0 = (R - 1) (q_1 \alpha_2 + q_2 \alpha_1 - k) - 4k \alpha_1 \alpha_2.$$  

(34)

This is eq. 113 in [24].

With the parameters of Fig. 10, the model predicts that the most unstable wavelength is 1.5 cm, to be compared with the 3 mm experimentally observed. As already seen with the scaling laws, we have to consider the finite size of the system: there is a cut-off on the wavelength when the height of one region ($d_1$ or $d_2$) is too small.

**G. Finite value of $d_2$**

For finite size systems, conditions must be imposed at the positions $d_1$ and $d_2$. As we have $d_1 \gg d_2$, we only consider the finite value of $d_2$. The precise condition at the boundary with the free meniscus is actually a complicated problem. The Laplace pressure becomes important close to the meniscus and the equations of motion used here fail. For sake of simplicity, we simply impose vanishing tangential and normal velocities. This leads to, at $z = d_2$:

$$A_2 e^{k d_2} + B_2 e^{-k d_2} + C_2 e^{q d_2} + D_2 e^{-q d_2} = 0$$

(35)

$$k A_2 e^{k d_2} - k B_2 e^{-k d_2} + q_2 C_2 e^{q d_2} - q_2 D_2 e^{-q d_2} = 0$$

(36)

**FIG. 12. Theoretical growth rate as a function of the experimental value shown in Fig. 5 (○), 6 (●) and 7 (×). The theoretical value is determined by numerical resolution of the system, using the experimental values of $d_2$, $h_2$ and $h_1$ (we used $h_2 = 1.5\mu m$ for the vertical series). The other parameters are $\rho = 10^3 \text{kg/m}^3$ and $\mu_s = 0.8 \times 10^{-4} \text{kg/s}$. The line corresponds to $n^{th} = n_e^{th}$.**

We thus obtain a system with 6 equations and 6 unknown eqs. 25-30 and 35-36. For a given set of physical parameters, and for a suited range of $k$, we determine numerically the value of $n(k)$ for which the system determinant vanishes. We adapt the range of $k$ to make sure that $n(k)$ has a maximum $n_{max}$ in that range. We solved the problem for the parameter sets $h_1 = 0.5\mu m$, $h_2 = 2\mu m$, $\mu_s = 10^{-4} \text{kg/s}$, for 3 values of the effective gravity, and for $d_2$ between 1 and 100 mm. The corresponding wavelengths $\lambda = 2\pi / k_{max}$ and characteristic times $\tau = 1 / n_{max}$ are shown in Fig. 11: as expected, the scaling laws previously established are recovered in the limit of small and large $d_2$.

**V. COMPARISON WITH THE EXPERIMENTAL DATA**

To compare our predictions with the experimental data, we took into account in the model, for each experimental data, the measured value of the parameters $h_1$, $h_2$, $d_2$, $g$. For the set up (A), the only unknown parameter is the interfacial shear viscosity $\mu_s$. For set-up (B) (without spectral camera), the thickness $h_1$ has been deduced from the comparison of the foam film color and a reference color chart with an uncertainty of ±100nm. The error bar on $h_2$ is larger, but counting the interference fringes indicates a value in the range $[1 - 2] \mu m$. The thickness $h_2 = 1.5\mu m$ has been used in the model. The comparison between the growth rates obtained theoretically and experimentally is shown in Fig. 12 and shows a reasonable agreement for $\mu_s = 0.8 \times 10^{-7} \text{kg s}^{-1}$, for both solutions. The order of magnitude of this viscosity is the expected one for the SDS solution. Indeed
\( \mu_s = 0.6 \pm 0.41 \times 10^{-7} \text{kg.s}^{-1} \) has been found in [23]. A slightly larger value was expected for the solution with dodecanol.

Then, using \( \mu_s = 0.81 \times 10^{-7} \text{kg.s}^{-1} \), the experimental data of Figs. 5, 6 and 7 can be compared to the scaling laws eqs. (10, 11) at large \( d_2 \) (solid lines in the figures) and eqs. (12, 13) at small \( d_2 \) (dashed lines in the figures). In Fig. 5 and 6, we observe no correlation between \( d_2 \) and \( \lambda \), which indicates that the large \( d_2 \) scaling should be used. Consistently, we show in Fig. 5 and 6 that the variations of \( \lambda \) and \( n \) are compatible respectively with the scalings \( g^{-1/3} \) (eq. 10) and \( g^{2/3} \) (eq. 11). Similarly, the wavelengths of Fig. 7 are compatible with a scaling as \( d_2 \) (eq. 12) at small \( d_2 \) and a constant value (eq. 10) at large \( d_2 \).

However, the prefactors used for these fits differs from the predicted ones. Especially the power law 10 is predicted to have a prefactor of the order of 20, whereas a prefactor of the order of 4 is systematically needed to fit the data of Figs. 5, 6 and 7. As \( \lambda \) is of the order of 2 or 3 times \( d_2 \), the finite size effect should be non negligible and the measured value for \( \lambda \) should be in contrast smaller than the asymptotic prediction of eq. 10, as shown in Fig. 11.

Finally all the measured wavelengths are plotted as a function of the full numerical prediction in Fig. 13. It confirms that the model is not fully quantitative and that the transition from the large to the small film height regime is not entirely understood.

As the predicted exponents of the film thicknesses and of the interfacial viscosity are small, the discrepancy can not be attributed to the uncertainty on these quantities. One possible reason for the overestimation of the wavelength by the model is that the system is not steady. The thick film height \( d_2 \) varies during the instability development and the observed wavelength may thus be the fastest mode associated to an earlier value of \( d_2 \). As \( \lambda_{th} \) increases with \( d_2 \), this effect is compatible with the observed overestimation. Another possibility is that the boundary condition at the top used in the model, i.e. a vanishing velocity, is not the actual one. The normal velocity at the top meniscus should be small, but the tangential velocity at the meniscus may be non negligible.

VI. CONCLUSION

In this paper, we describe quantitatively for the first time the Rayleigh Taylor instability arising in foam film when a thick film is produced on top of a thin film. As this situation is induced by any extension of a non-horizontal film, it should also happen in a 3D foam when local deformations are induced by coarsening or external shear. This process should potentially influence the film thickness distribution in 3D foams, and thus the gas diffusion from one bubble to the other and so the coarsening rate, or the energy dissipation induced by film shearing, and thus the apparent viscosity. It is also an original ex-

![Graph showing theoretical wavelength as a function of the experimental value shown in Fig. 5 (○), 6 (●) and 7 (■) with the same parameters as in Fig. 12. (Top) The color chart shows the effective gravity value in m/s². Red corresponding to 9.8 m/s². (Bottom) The color chart shows the Frankel’s film height \( d_2 \) in mm.](image-url)
APPENDIX : NORMAL STRESS CONTINUITY

The normal stress in each domain is \( K = 2\mu_s\partial_z w_i + \partial_z \sigma + \delta\sigma_i \), so

\[
k^2 K = 2k^2 \mu_s \partial_z w_i + \frac{\rho gh_1 k^2}{n} w_i + \rho h_i n \partial_z w_i + \mu_s \left(k^2 - \partial_{zz}\right) \partial_z w_i = \frac{\rho gh_1 k^2}{n} (A_i + B_i + C_i + D_i) + (3k^2 \mu_s + \rho h_i n) (k A_i - k B_i + q_i C_i - q_i D_i) - \mu_s (k^3 A_i - k^3 B_i + q^3_i C_i - q^3_i D_i) \tag{37}
\]

The condition 28 is thus, using \( \mu_s(q_i^2 - k^2) = \rho h_n \) in each phase,

\[
0 = A_1 \left( \frac{\rho gh_1 k^2}{n} + 2k^3 \mu_s + \rho h_1 n k \right) + B_1 \left( \frac{\rho gh_1 k^2}{n} - 2k^3 \mu_{s,1} - \rho h_1 n k \right) + \\
C_1 \left( \frac{\rho gh_1 k^2}{n} + 2k^2 q_1 \mu_s \right) + D_1 \left( \frac{\rho gh_1 k^2}{n} - 2k^2 q_1 \mu_s \right) + \\
-A_2 \left( \frac{\rho gh_2 k^2}{n} + 2k^3 \mu_s + \rho h_2 n k \right) - B_2 \left( \frac{\rho gh_2 k^2}{n} - 2k^3 \mu_s - \rho h_2 n k \right) + \\
-C_2 \left( \frac{\rho gh_2 k^2}{n} + 2k^2 q_2 \mu_s \right) - D_2 \left( \frac{\rho gh_2 k^2}{n} - 2k^2 q_2 \mu_s \right) \tag{38}
\]

Making finally the transformation (38) - \( \frac{-\rho gh^2/(2n)(h_1 + h_2)}{2} \) (25) - \( 2k^2 \mu_s(26) \) we get eq. 30.

APPENDIX 2

The determinant of the system 33 can be reduced by the transformation \( C_2 \to C_2 - C_1 \), \( C_3 \to C_3 + C_1 \) and \( C_4 \to C_4 - C_3 \), leading to

\[
\begin{vmatrix}
1 & 0 & 0 \\
k & q_1 - k & 0 \\
2k^2 \mu_s & \mu_s(q_1^2 - k^2) & 0 \\
2k & q_2 - k \\
\end{vmatrix} = X \begin{vmatrix} 1 & 0 & 0 \\ \alpha_1 & R - (\alpha_1 + \alpha_2) & \alpha_2 \\ \alpha_1 & R - 1 & \alpha_2 \end{vmatrix}
\]

\[
\begin{vmatrix} q_1 - k & 2k & q_2 - k \\ \rho k_1 n & 2k & -\rho k_2 n \\ \alpha_1 & R - 1 & \alpha_2 \end{vmatrix}
\]

\[
\alpha_1 = \frac{q_1 - k}{\mu_s(q_1^2 - k^2)} - \frac{2k}{\mu_s(q_1^2 - k^2)} + \frac{q_2 - k}{\mu_s(q_1^2 - k^2)}
\]

The condition 34 is finally obtained by a development of the determinant along the last line.

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Appendix B
Dynamical coupling between connected foam films: interface transfer across the menisci

Adrien Bussonnière, Evgenia Shabalina, Xavier Ah-Thon, Mickaël Le Fur, and Isabelle Cantat
Univ Rennes, CNRS, IPR (Institut de Physique de Rennes) - UMR 6251, F- 35000 Rennes.
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The highly confined flow of the liquid phase, trapped between the gas bubbles, is at the origin of the large effective viscosity of the liquid foams. Despite the industrial relevance of this complex fluid, the foam viscosity remains difficult to predict, because of the lack of flow characterization at the bubble scale. Using an original deformable frame, we provide the first experimental evidence of the interface transfer between a compressed film (resp. a stretched film) and its first neighbour, across their common meniscus. We measure this transfer velocity, which is a key boundary condition for local flows in foams. We also show the dramatic film thickness variation induced by this interface transfer, which may play an important role in the film thickness distribution of a 3D foam sample.

Dry liquid foams behave as shear thinning complex fluids, having a surprisingly high effective viscosity [1]. The macroscopic deformations turn at local scale into bubble deformations, thin soap film shearing or stretching, thin film extraction from the menisci, and rapid bubble neighbour switching, called T1. Foam coarsening, or collapse, induce similar local motions. The associated dissipation rates have been identified in the seminal article of Buzzà et al. [2], but the determination of the relative importance of these various dynamical processes remains an open problem, which motivated numerous experimental and theoretical studies at the scale of few foam films. In these studies, the film deformations are imposed either by moving the system boundaries [3–8] or by forcing T1 transformations [9–16], and, in both cases, the models used to rationalised the observations rely on a strong assumption on a precise quantity: the surfactant surfacic face tension of deformed foam (few mN/m), the surface concentration \( \Gamma \) and the surfactant interfacial concentration and \( v_s \) the interfacial velocity at the meniscus. A generic situation is shown in Fig. 1, corresponding to the case addressed in [9].

This crucial parameter \( j_s \) controls the coupling between one film to another in all kinds of foam deformation involving film area variations. As the variations of surface tension of deformed foam films are relatively small (few mN/m), the surface concentration \( \Gamma \) remains close to its equilibrium value \( \Gamma_{eq} \) and the flux \( \Gamma v_s \) is mainly governed by the interfacial velocity. In the literature, several phenomenological rules are proposed: (i) the full transfer, proposed in [9] and imposing \( v_s = dL/dt \), with \( dL/dt \) the film length variation defined in Fig. 1; (ii) the absence of flux with \( v_s = 0 \) [12]; or (iii) an ad-hoc law, between the two previous limiting cases [3, 13].

This paper provides the first experimental determination of \( v_s \), on both sides of a meniscus. We show that (i) for the chosen deformation and foaming solution, \( v_s \) has comparable values on both sides of the menisci. Identifying \( j_s \) with \( \Gamma_{eq} v_s \), this indicates that the meniscus does not play any significant role of surfactant reservoir for the interface ; (ii) the obtained value for \( v_s \) is smaller than the film deformation rates, but of the same order. These results evidence the important coupling between adjacent foam films and the feasibility, and the need, of a systematic measure of \( j_s \), which appears as an unavoidable preliminary step to improve local models of foam rheology.

![Fig. 1. Scheme of the interfacial flow in the vicinity of a meniscus, driven, for exemple, by the decrease of the size L of the lateral films.](image)

The menisci motions inherent to previously studied T1 deformations [9, 15] or the film curvature in the catenoid geometry [3, 5] make laborious the quantification of the interface transfer velocity. To overcome these difficulties, we design a dedicated frame, shown in Fig 2a, on which a horizontal central film is connected to four lateral films, labelled by the index \( i \in \{1;4\} \), whose lengths are controlled by linear piezo motors. This geometry has been chosen so that each film remains flat and stay in the same plane during the deformation.

The lateral films make an angle of 120° with the central film, as prescribed by the two X-shaped metallic pieces constituting the lateral boundaries of the system (see Fig. 2a). We impose a deformation between an initial shape and its mirror image (see Fig. 2b,c): the distance \( d_i(t) \) between the moving external edge \( i \) and the corresponding meniscus is \( d_{1,2} = d_e = d - Ut \) on the left side and \( d_{3,4} = d_r = d + U(t - \tau) \) on the right side, between \( t = 0 \) and \( t = \tau \). The four motors stop after a displacement of 10 mm, at time \( \tau = 0.2 \) s. The motor velocity is \( U = 50 \)
FIG. 2. (a) Scheme of the set-up. The 5 foam films and the two free menisci are represented (in blue), as well as the front part of the solid frame (X-shape piece, in red). The motion of the system in the (x,z) plane, at the time \( t = 0 \). The 4 moving edges (top and bottom left \( t_l \), \( b_l \), and top and bottom right \( t_r \), \( b_r \)) are shown in red. (c) Same system, after motor motion. The thick parts of the central and right films are the pieces of film extracted from the meniscus.

mm/s and \( d = 17.1 \) mm. Syringes positioned along the four upper arms of the X-shaped pieces each injects a flux \( Q = 0.05 \) mL/min of surfactant solution in the system. This allows to prepare a reproducible initial state for the films and menisci, which are deformed 15 s after being pulled at a controlled velocity out of the foaming solution.

The solution is made of deionised water, sodium dodecyl sulfate (SDS, cmc = 2.36 g/L) at 5.6 g/L, dodecanol at 0.05 g/L, glycerol at 15 \% in volume and fluorescein at 0.8 g/L. The viscosity is \( \eta = 1.52 \) mPa.s, and the equilibrium surface tension is \( \gamma = 32 \) mN/m.

The dynamical behavior is fully characterized by simultaneously measuring the film thickness and velocity. The film thickness \( h \) is measured on a line \( L \) oriented perpendicular to the free menisci (see Fig. 2a), with a spectral camera \( (C_1) \) Resonon Pika L (see Supp. Mat. A [17]) illuminated with a halogen light. The velocity field is deduced by the motion of photobleached spot, monitored with the camera \( C_2 \). A 488 nm laser is split in two beams; one is expanded to illuminate a large part of the film; and the other is focused during 15 ms, prior to the motor motion, to locally photobleach the fluorescein (see [18] for more details). This technic is well suited to probe film motion since the velocity is uniform over the film thickness in flat film due to (i) the uniformity of the pressure preventing any Poiseuille flow and to (ii) the symmetry of the system and of the imposed deformations implying identical interface velocities and no shear (far enough from the menisci, see Fig. 1). Moreover the experimental time is shorter than the fluorescein diffusion time. Consequently, the spot remains well contrasted along its whole trajectory and is a good passive tracer.

The camera \( C_2 \) is also used to track the positions \( x_l(t) \) and \( x_r(t) \) of the left and right menisci, respectively. They slightly move during the deformation, without modifying significantly the central film area. In the central film, we denote by \( s \) the distance to the left meniscus, i.e. \( s = x - x_l(t) \), while in the lateral films \( s \) is the distance to the corresponding free meniscus (see Fig. 2c).

Finally, the vertical dimension \( w_m \) of the left meniscus is measured before deformation by light transmission using a collimated white light and a telecentric objective Edmund optic Silver-TL 4x (camera \( C_3 \)). The vertical position of the meniscus remains almost constant, with variations less than 100 \( \mu \)m.

The thickness profiles during and after motor motion are shown in Fig. 3 for different times, in the central, left and right films, as well as the volumes (per unit length in the y direction). These plots evidence the following phenomenology:

(i) Initial state \((t < 0)\). The central film thickness is uniform and equal to \( h_0 = 400 \pm 200 \) nm. The left film is 3 \( \mu \)m thick and the right film is 1.5 \( \mu \)m thick. These last thicknesses have steady but out of equilibrium values, which depend on the film size and inclination, and on the injected flux. The meniscus radius of curvature is deduced from the equilibrium relationship \( r_m = w_m = 320 \mu \)m [1].

(ii) Lateral films stretching and compression \((t \in [0 - 0.1] \) s\). The right film get thinner and the left film get thicker, but their volumes remains constant. This evidences a homogeneous films deformation with no exchange with the menisci. No noticeable evolution is observed in the central film.

(iii) Interface transfer across menisci \((t \in [0.1 - 1.5] \) s\). In the right film, thick pieces of film are extracted from both the right and bottom right menisci (see Fig. 2c). A straight, well defined, frontier separates these extracted films from the film initially present (hereafter called the initial film). These steps (ii) and (iii) was already observed in a stretched film in [5] and in [15]. Similarly, in the compressed film (left), the film volume decreases, evidencing a liquid flux toward the menisci (see Fig. 3a, inset). In the central film, a thick film is extracted from the left meniscus and the thin film initially present flows into the right meniscus. This motion happens without
any deformation of the film, which size and position remain constant, and is thus solely induced by the interface transfer from the adjacent films.

(iv) Eventually, the system recovers its slow drainage behavior, after a delay of the order of 10 s.

Note that the top/bottom symmetry of the motion in the right film is broken at \( t \approx 0.25 \) s by a gravitational instability which imposes a stratification of the film thickness, thicker parts of the film laying below the thinner parts. A single extracted film is thus observed at later times in the right film, close to the bottom meniscus. Some film is however continuously extracted at the top, but it falls down after its extraction from the meniscus and merges with the thick film produced at the bottom (see [19]).

The motion of the central film is quantified from the velocity measurements. We measured the photo-bleached spots position relatively to the left meniscus \( s(t) = x(t) - x_{l}(t) \). The displacement of the spots in the \( y \) direction remains smaller than 10 % of the total spot displacement, except for positions very close from the right meniscus, where we observe marginal regeneration which locally breaks the invariance in the \( y \) direction [20]. In the following, the spots at a distance smaller than 1 mm from the right meniscus are discarded. We recorded 4 trajectories \( s_{i}(t) \), each being averaged over 10 experiments. The index \( i \in [1-4] \) refers to the initial distance \( s_{i}(0) \) between the spots (i) and the left meniscus, ranging from 1 to 4 mm.

These trajectories evidence a translational motion of the central film, with almost no deformation, over a distance which is a significant fraction of the motor displacement. Fig. 4(inset) shows the distance between the spots [(2)] \( -(4) \] and the spots (1) as a function of time. The spots (1) and (2) are close to the left meniscus, and the distance between them is almost conserved during the whole motion, showing the absence of film deformation in this region. The distances between spots (3,4) and spots (1) slightly decrease, indicating a small film compression in the right part of the film. However, no clear signature of this compression is observable in the thickness map (Fig. 3b), and residual recirculations close to the right meniscus may be at the origin of this small drift, especially at the longest times.

The trajectory of spots (1) relatively to their initial position, i.e. the distance \( L^{c}(t) = s_{1}(t) - s_{1}(0) \), is plotted in Fig. 4. As the central film moves as a whole close to the left meniscus, \( L^{c} \) corresponds to the amount of film extracted from the left meniscus. If we neglect the small compression observed close to the right meniscus, it corresponds also to the amount of film absorbed by the right meniscus.

We now compare this transferred length \( L^{c}(t) \) to the amount of interface \( L^{f}(t) \) going from the compressed film to the left meniscus, and to the amount \( L^{r}(t) \) going from the right meniscus to the stretched film.

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**FIG. 3.** Film thickness as a function of \( s \) (defined in Fig. 2c), in the bottom left film (a), the central film (b) and the bottom right film (c), at times \( t_{1} = 0 \) s (step i); \( t_{2} = 0.04 \) s (step ii); \( t_{3} = 0.075 \) s (step iii, before the gravitational instability) and \( t_{4,5} = [0.6, 1.25] \) s (step iii, after instability). The solid lines represent the average of 5 experiments for the lateral films and 50 for the central, the shaded areas are the standard deviation. The dashed lines in (b) correspond to the thickness profiles predicted by eq. 2. The insets show the left (a) and right (c) film volumes (per unit length in the \( y \) direction) \( V_{l} \) and \( V_{r} \), and indicates the times \( t_{i} \), time domains (1 - iv) and motor motion time (shaded red).
As the direct velocity measurements from the photo-bleached trajectories are not feasible because of the gravitational instabilities, these quantities are determined from the thickness profile only, at the price of an additional assumption: we assume that the same amount of interface goes in (or out) the top and bottom meniscus bounding the lateral film. In that case, we establish in Supp. Mat. B [17] that:

\[ v_{r,t}^{sym}(0, t) = \frac{1}{2} \left( \frac{d_r}{r} \right) \left( \frac{d_r}{r} \right) \],

with \( v_{r,t}^{sym}(0, t) \) the film velocity at the free meniscus and \( \frac{d_r}{r} = \frac{\langle h_r \rangle_{(t)} - \langle h_r \rangle_{(t)}}{\langle h_r \rangle_{(t)}} \) the film extension in the right and left films, respectively. The spatial average \( \langle h_r \rangle \) is made respectively on the whole left film, and on the thin part of the right film. Then we define \( L^c_{sym}(t) = \int_0^t \int_0^t v_{r,t}^{sym}(0, t) dt \) and \( L^c_{sym}(t) = -\int_0^t \int_0^t v_{r,t}^{sym}(0, t) dt \).

In the right film, the top and bottom extracted films have the same width before the instability begins, thus confirming the validity of our symmetry assumption, at least at short time. In the left film, a spot trajectory \( s_\alpha(t) \) has been obtained, again at short time, allowing to compute \( v_t \) as \( v_t(0, t) = \dot{s}_\alpha - \frac{x}{2} \dot{s}_\alpha \), which does not requires any symmetry assumption (see Supp. Mat. B [17]). The prediction \( L^l(t) = -\int_0^t v_t(0, t) dt \) is in qualitative agreement with \( L_{sym}(t) \), as shown in Fig. 4.

The quantity \( L^c_{sym} \) and \( L^c_{sym} \) show a behavior very similar to \( L^c \) with time (Fig. 4). A first conclusion is thus that the motor displacements lead to the transfer of interface from the left film to the right film across the two free menisci and the central film, occurring with negligible in-plane deformation, and with some delay with respect to the motor motion. The amount of interface transferred after 1 s is of the order of 3 mm, which is a significant part of the 10 mm displacement of the motor.

This interface transfer across the central film is coupled to the extraction of thick film from the left meniscus. It is separated from the thin film by a sharp frontier, moving at the velocity \( v_c(t) = dL^c/dt \) (see Fig. 4 inset).

The thickness of the film extracted from a meniscus of radius \( r_m \), at a constant velocity \( v \) is given by the Frankel’s expression \( h_{fr} = 2.66 r_m (\eta v / \gamma)^{2/3} \) [20]. In the central film, the dynamical meniscus lateral extension scales as \( \delta_{sym} = r_m (\eta v / \gamma)^{1/3} \sim 10 \mu m \) and the film goes through this region in a time \( t_{dyn} \sim \delta_{sym} / v_c \sim 10^{-3} \). As \( t_{dyn} \ll \tau = 0.2 s \), the time scale of the velocity variations (see Fig. 4), an approximation of quasi steady motion is justified. The thickness gradient at the frontier is of the order of \( \phi \approx 10^{-2} \) and it decreases on the typical time scale \( \phi^{-4} \eta h / \gamma \approx 10 s \ll \tau \) [21]: once extracted from the meniscus, the film thus simply follows a passive convection. Moreover, as the central film extension rate is negligible, it moves without any deformation. As a consequence, the film at the position \( s \) at time \( t \) has been extracted from the meniscus at time \( t_0(s) \) implicitly given by the relation \( s = \int_0^t v_c(t) dt \), and the theoretical prediction for its thickness is

\[ h_{th}(s, t) = 2.66 r_m (\eta v_c(t_0(s)) / \gamma)^{2/3} \] (2).

This theoretical profile is plotted in Fig. 3b (dashed lines) and shows a quantitative agreement with the experimental data, which validates a posteriori the assumptions made.

In conclusion, we evidence and quantify interface transfer between adjacent foam films which constitutes a first critical step toward more realistic foam models taking into account the coupling between films. Moreover, the imposed deformation induces a thickness increase by a factor of 10 in a film which is only a neighbor of the deformed films. The thickness relaxation toward its initial value lasts for several seconds. As a consequence, the localized deformations occurring in 3D foam samples, due to aging or global flow, may redistribute the liquid phase in the thin films at a much larger distance from the local deformation than previously expected. This process is potentially an efficient factor of rejuvenation of the foam films, competing with the slow drainage imposed by the meniscus capillary suction and gravity. The original setup we developed proved, on the example of the imposed deformation we choose, its efficiency to identify deformation modes in foam films, and to measure them with an accuracy allowing for a quantitative comparison with local theoretical models. We believe this should remains the case for a large class of imposed deformations, which paves the way to a deeper understanding of dissipation in foams.
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Supplemental Material: Dynamical coupling between connected foam films: interface transfer across the menisci

Adrien Bussonnière, Evgenia Shabalina, Xavier Ah-Thon, Mickaël Le Fur, and Isabelle Cantat
Univ Rennes, CNRS, IPR (Institut de Physique de Rennes) - UMR 6251, F-35000 Rennes.
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In this document, we provide more details on the spectral camera operating and described the methods used to extract the interface lengths transmitted to the menisci.

A - SPECTRAL CAMERA

The spectral camera \(C_1\) makes the image of \(L\) and measures the spectrum \(I(\lambda)\) of the light reflected by each point of \(L\). Each point of \(L\) corresponds to a different sensor line: the intensity of the light of wavelength \(\lambda_j \pm 0.55\) nm, in the range \([375 - 1010]\) nm, reflected by the piece of film at the position \(x_i\) along \(L\) and of size \(dx\ dy\) is recorded by the pixel \((i, j)\) of the 2D sensor. The distance \(dy\) is fixed by the width of a slit in the camera and \(dx = 14.9\) \(\mu\)m is the spatial resolution. The light trajectory is in the plane \((y, z)\) and the incidence angle on the film is \(\theta = 45^\circ\) for the central film and \(58.5^\circ\) for the lateral films. Finally, the thickness \(h(x, t)\) of the line \(L\) is extracted by using the relation

\[
I(\lambda) \propto 1 - \cos \left( \frac{4\pi hn}{\lambda} \left(1 - \frac{\sin^2 \theta}{n^2} \right)^{1/2} \right), \tag{1}
\]

with \(n = 1.33\) the water refractive index.

B - TRANSMITTED INTERFACE LENGTH

In this section, we establish the expression used to compute the amount of interface transmitted across the different menisci. It can be either deduced directly from the velocity map, or, at the price of additional assumptions discussed here, from the thickness map.

Uniformity of the lateral film extension

The aim of this section is to validate the assumption of uniform extension/compression in the lateral films, especially in the stretched film which has a very heterogeneous thickness.

In the thin films, the velocity is homogeneous across the film thickness. A piece of film of area \(dS(t)\) at the position \(M(t)\) is thus a closed material system of constant volume \(dV\). We define the area dilatation, relatively an arbitrary reference at \(t = 0\), as \(\varepsilon(M, t) = dS(t)/dS(0)\). This quantity is related to the film thickness profile. Indeed, the mass conservation imposes \(dV = dS(t)\ h(M, t)\) at each time, which leads to the general formula

\[
h(M, t)\ \varepsilon(M, t) = h(M_0, 0) \quad \frac{dS(t)}{\varepsilon(M, t)} = dS(0) \tag{2}
\]

with \((M_0, 0)\) and \((M, t)\) two points on the trajectory of the same material point.

In the lateral films, one important difficulty is that the piece of film trajectories are unknown, because the gravity-induced stratification breaks the invariance of the velocity field in the \(y\) direction. At each time, some new films are produced at the top and bottom menisci and flow to their final position in the stratification, keeping the same area. However, as the motion toward the final position is fast, the stratification process itself insures the invariance by translation along the \(y\) direction of the thickness field: the part of film having a certain thickness is an thin horizontal band. The relevant quantity to address the dynamics is thus \(A(h, t)\): the total length of film, measured in the \(s\) direction, having a thickness smaller than \(h\) at \(t\). This length is a film area per unit length in the \(y\) direction. The graph of \(h\) as a function of \(A(h)\) coincides with the graph of the actual film profile \(h\) as function of \(s\) if the gravity-induced stratification...
already happened (see times $t_4$ and $t_5$ in Fig. 4c of the main paper).

This quantity $A(h, t)$, schematized in Fig. 1, does not involve spatial information anymore. In Fig. 1(a), we represent schematically the thickness profile of a stretched film, at a time $t$. The blue parts represent the initial film; the red parts represent the Frankel’s film produced at the earlier times; it was produced at the highest velocity and has thus the highest thickness; the pink parts represent the Frankel’s film extracted just before $t$: it is closer to the menisci and has a smaller thickness than the red part. Fig. 1(b) shows $h$ as a function of $A(h)$ for the same film, and the contribution of each part of the film can be followed using the color code, each color being associated to a material domain: the thinnest parts of the film are represented at the left end, and the thickest parts at the right.

The Fig. 1(c) shows $h$ as a function of $A(h)$ at the time $t + dt$. The light pink part is the Frankel’s film extracted between the times $t$ and $t + dt$. Its thickness is smaller than in the dark pink region of Fig. 1(a), as the extraction velocity decreases with time. Plotted as a function of $A(h)$, it thus comes between the blue and the dark pink domains. Moreover, during the delay $dt$, the whole film is compressed: in the blue, red and dark pink domains, the thicknesses increase and the areas decrease. The shape of these domains at $t + dt$ can be deduced from the domain shapes at $t$ by a proper rescaling, based on $\varepsilon$. The light pink domain, in contrast, has no corresponding domain at $t$.

To summarize, the signature of a homogeneous compression or dilatation of a stretched film is that the left part and the right part of the curves $(h, A(h))$ are properly rescaled by eq. 2 (the variable $A$ replacing the variable $dS$), using the same value of $\varepsilon$. In the following, we use this criteria to evidence that the Frankel’s film and the initial film have the same compression or dilatation $\varepsilon$.

The successive thickness profiles investigated are shown in Fig. 4c of the main paper. They are shown in the $(h, A(h))$ representation in Fig. 2a.

We first focus on the thin part of the film, at the left part of the curve, corresponding to the film initially present. It is separated from the Frankel’s film by a well defined frontier, at the position $A_f$ where the thickness suddenly increases, and is almost flat. Its extension can be measured from the thickness $\langle h_{r, in} \rangle$, averaged over the whole initial film, as

$$\varepsilon_r(t) = \frac{\langle h_{r, in} \rangle(t=0)}{\langle h_{r, in} \rangle(t)}.$$  \hspace{1cm} (3)

This extension is plotted as a function of time in the inset of Fig. 2a.

In Fig. 2b we plot $h\varepsilon_r$ as a function of $A/\varepsilon_r$ for the thickness range corresponding to the initial film (i. e. for $A \leq A_f$ or slightly above, so that the frontier is visible). The thickness is of course well rescaled, by definition of $\varepsilon_r$ (the curves are slightly translated vertically in the figure for sake of clarity). The important point is that $A_f(t)/\varepsilon_r(t)$ has a steady value, which confirms that the initial film volume is conserved and that the extension in the $y$ direction is negligible.

We actually observe a Frankel’s film extraction from the menisci on the side of the setup (in contact with X-shape frame), of the same order of magnitude as from the two other menisci, which may lead to a contraction of the right film in the $y$ direction (at least before the inset of the gravitational instability). However, the size length (7.1 mm at the end of the deformation) is much smaller than the width of the setup (41 mm), making this effect negligible.

We now address the right part of the curve in Fig. 2a, and we show that the top and bottom Frankel’s films, once extracted from the menisci, follow the same deformation as the initial film, despite the thickness contrast.

The Frankel’s films are not closed systems, as they are...
the rescaling used for the thin part of the film should also work for the highest thicknesses, but not for the intermediate ones, which correspond to new film added in the system.

To follow the behavior of these pieces of film of largest thicknesses at different times, we plot in Fig. 2c $\frac{\Delta L_{\text{trans}}}{h_\text{max} - h}$ as a function of $(A(h_{\text{max}}) - A)/\varepsilon_r$, i.e. the amount of film of thickness larger than $h$ (the largest thicknesses thus correspond now to the left part of the curve). The parts of the curves which are properly superimposed by this rescaling are plotted with solid lines, whereas the remaining parts are in dashed lines. Apart from the small peak at short time that disappears at larger times, being probably absorbed by the meniscus, the left part of the curves are well rescaled. This allows to conclude that the extension rate is the same in the thin part of the film and in the Frankel’s films. The compression of the whole right film is thus given by $\varepsilon_r$, defined in eq. 3.

Finally, in the compressed film, the thickness is almost uniform, equal to its mean value $\langle h_t \rangle(t)$. As the different pieces of film can only leave the film, a piece of film of thickness $\langle h_t \rangle(t)$ at $t$ was in the film at $t = 0$ and had the thickness $\langle h_t \rangle(t=0)$. On the basis of eq. 2, we thus define

$$\varepsilon_t = \frac{\langle h_t \rangle(t=0)}{\langle h_t \rangle(t)}.$$  

(4)

**Velocity field in the films**

We consider a thin film bounded by two menisci located at $s = 0$ and $s = d$ experiencing a homogeneous compression/extension characterized by $\varepsilon$. The velocities and positions are taken relatively to the meniscus located at $s = 0$ and the velocity field can be written as, from eq. 2(right):

$$v(s, t) = \frac{\dot{\varepsilon}}{\varepsilon} s + v(0, t).$$  

(5)

The aim is to determine $v(0, t)$ to compute the transmitted length as

$$L = \left| \int_0^t v(0, t) \, dt \right|.$$  

(6)

The velocity at which the interface leaves the film at $d$ is $v(d, t) = \dot{d}$. Rewriting eq. 5 at the position $s = d$, we get

$$\dot{d} = \dot{\varepsilon} - v(0, t) - (v(d, t) - \dot{d})$$  

(7)

In case of symmetrical behavior of the two menisci, $v(0, t) = -(v(0, t) - \dot{d})$ leading to

$$v(0, t) = \frac{1}{2} \left( \dot{d} - \frac{\dot{\varepsilon}}{\varepsilon} \right).$$  

(8)

**Transmitted length in the central film**

In the central film, $\dot{\varepsilon}_c \approx 0$. In this case, using equation 5, the velocities $v_c(0, t)$ (left meniscus) and $v_c(d_c, t)$ (right meniscus) are equal to the spot velocity $s_\alpha$. The transmitted interface length is then determined as:

$$L_f^c(t) = \int_0^t \dot{s}_1(\tau) d\tau = s_1(t) - s_1(0).$$  

(9)

**Transmitted length in the right film**

In the right film, we determine the extension $\varepsilon_r$ from eq. 3 and use it in eq. 8. The assumption of symmetrical extraction can be directly verified on the thickness map for $t < 0.25$ s, before gravitational instability is set, whereas it must be assumed at larger times. The expression for $L_{\text{sym}}^r(t)$ is finally

$$L_{\text{sym}}^r(t) = \int_0^t \left( \dot{d}_r(\tau) - \frac{\dot{\varepsilon}_r(\tau)}{\varepsilon_r(\tau)} d_r(\tau) \right) d\tau.$$  

(10)

**Transmitted length in the left film**

In the left film, a photobleached spot is used as a tracer of the velocity for $t < 0.2$ s, time at which the photobleached spot trajectories become non reproducible. From eq. 5, we get

$$v_l(0, t) = \dot{s}_\alpha - \frac{\dot{\varepsilon}_l}{\varepsilon_l} s_\alpha.$$  

(11)

The transmitted interface length is then calculated using

$$L_f^l = -\int_0^t v_l(0, \tau) d\tau.$$  

(12)

An alternative for longer times is again to assume a symmetrical absorption velocity in the left and bottom left menisci, i.e. $v_l(0, t) = \dot{d}_l - v_l(d_l, t)$. In this case, using equation 8, we get

$$v_{\text{sym}}^l(0, t) = \frac{1}{2} \left( \dot{d}_l - \dot{d}_l \frac{\dot{\varepsilon}_l}{\varepsilon_l} \right),$$  

(13)

with $\varepsilon_l$ given by eq. 4. The transmitted interface length is:

$$L_{\text{sym}}^l = -\int_0^t v_{\text{sym}}^l(0, \tau) d\tau.$$  

(14)
Appendix C

Resumé

Les mousses liquides sont des dispersions concentrées de bulles de gaz dans une solution de surfactant. Elles possèdent des propriétés spécifiques largement utilisées dans l’industrie: production alimentaire et cosmétique, fluides de forage dans la production de pétrole, agents extincteurs du feu et solvants polaires. Les mousses ont des propriétés mécaniques qui peuvent être soit solides soit liquides, et font l’objet de nombreuses recherches [30]. Elles démontrent un comportement quasi-élastique jusqu’à la contrainte ou la déformation seuil, puis s’écoulent comme des fluides rhéofluidifiant. L’origine de la réponse élastique provient de la variation de la surface totale du film sous déformation.


Ces deux processus contribuent à la dissipation globale de la mousse sous cisaillage, et le principal problème consiste à déterminer l’importance de chacun de ces processus, et les dissipations associées. Dans ce but, nous utilisons quelques films et essayons de décrire leur comportement sous déformation. Les processus observés localement devraient être utiles pour construire un modèle de la viscosité effective des mousses 3D.

La thèse comprend trois parties et cinq chapitres ainsi que l’introduction, la conclusion et deux annexes avec les résultats récemment publiés de notre équipe.
La première partie, intitulée "Basics", comprend deux chapitres. L’objectif de cette partie est d’introduire les concepts de base des films de savon et les outils utilisés pour nos expériences.

Le chapitre 1 explique les informations générales sur les films de savon et fournit une revue de la littérature sur les études récentes sur la déformation des films de savon et la compréhension des propriétés des "films de Frankel". Au cours des expériences, nous avons utilisé plusieurs dispositifs optiques. Un paragraphe du premier chapitre est donc consacré à certaines informations optiques.

Le chapitre 2 explique les techniques permettant de créer et de déformer un film de savon et les outils optiques permettant de l’observer. Premièrement, nous présentons le montage nous permettant de créer trois films de savon reliés par un même ménisque. L’idée et la conception de l’installation ont été développées et entièrement réalisées dans notre équipe. Une section du chapitre 2 présente également la chimie utilisée dans les expériences.

La deuxième partie contient deux chapitres expliquant les expériences réalisées avec trois films de savon. Tout d’abord, nous étudions un film sous étirement au chapitre 3. Nous construisons un modèle pour expliquer le comportement visco-élastique du film étiré, testons le modèle sur nos données expérimentales et comparons les résultats prédits par le modèle et obtenus à partir d’expériences. Afin de déterminer l’épaisseur du film extrait, nous utilisons une caméra hyperspectrale, qui n’avait pas été utilisée auparavant par d’autres chercheurs.

Au chapitre 4, nous décrivons pour la première fois une instabilité 2D de Rayleigh-Taylor, qui se produit dans un film de savon sous étirement lorsqu’un film plus épais est produit pardi-dessus un film mince. Nous présentons un ensemble d’expériences avec plusieurs paramètres de contrôle tels que la gravité effective et la vitesse d’étirement du film. À la fin du chapitre, nous comparons un modèle linéaire aux données expérimentales. Sur la base de ce chapitre, un article a été soumis.

La troisième partie ne comporte qu’un chapitre, mais elle doit être bien séparée des précédentes car elle concerne un montage expérimental plus complexe. Au chapitre 5, nous développons et testons un montage original de cinq films de savon et de deux ménisques, qui a été breveté. En utilisant cette configuration, nous fournissons des résultats préliminaires mettant en évidence l’influence dynamique d’un film étiré sur ses voisins. La configuration a ensuite été utilisée par A. Bussonnière, alors que je me concentrais sur l’instabilité décrite au chapitre 4. Je suis co-auteur de l’article soumis sur le chapitre 5.
Conclusion


Au chapitre 1, nous avons présenté le problème de la déformation des films de savon et de l’extraction du ménisque d’un nouveau film plus épais, obéissant quantitativement à la loi de Frankel (en supposant des interfaces incompressibles).

Au chapitre 2, nous avons expliqué la structure du montage permettant de créer trois films de savon reliés par un ménisque commun. La conception et le développement de l’installation ont été réalisés par notre équipe. La première version, qui est utilisée pour toutes mes expériences, est en plastique, mais récemment, nous l’avons améliorée et avons construit une nouvelle version en métal, plus rigide et plus précise. Toutes les expériences ultérieures seront effectuées sur une nouvelle configuration. Dans ce chapitre, nous avons également expliqué la méthode d’utilisation de la caméra hyperspectrale pour les mesures d’épaisseur de film, ce qui nous permet de mieux comprendre le comportement de l’épaisseur de l’ensemble du film par rapport aux résultats obtenus avec un spectromètre habituellement utilisé pour ce type de mesure.


Dans le même chapitre, nous avons produit une carte d’épaisseur $h(z, t)$ et nous avons essayé de comparer les résultats avec la loi de Frankel. Nous avons trouvé une différence d’épaisseur (la valeur prédite est plus grande pendant l’extension et plus petite par la suite).
Pendant l'extension, il est possible que nous n’atteignions pas un régime stationnaire, ce qui peut expliquer une plus petite épaisseur au début et nous avions beaucoup d’informations perdues lors de l’enregistrement du spectre par la caméra hyperspectrale, ce qui pourrait entraîner une différence d’épaisseur par la suite. Comme il a été utilisé et testé pour la première fois, le protocole expérimental avec la caméra hyperspectrale devrait être amélioré pour obtenir de meilleures informations spectrales.

Au chapitre 4, nous avons présenté et décrit un exemple d’instabilité 2D de Rayleigh-Taylor, qui se produit dans un film de savon étiré lorsque le film extrait le plus épais se trouve au sommet du film le plus fin. Comme cela n’était pas décrit précédemment, nous avons développé et expliqué un modèle linéaire du problème. Dans cette thèse, j’ai d’abord examiné le cas de la taille illimitée du film extrait \(d_2\) et comparé les valeurs de la longueur d’onde de l’instabilité \(\lambda\) et du taux de croissance \(n\) à des valeurs expérimentales. Nous avons effectué une grande série d’expériences en essayant d’ajuster plusieurs paramètres expérimentaux tels que \(\bar{g}\) la gravité effective et \(U\) la vitesse d’étirement. Nous avons découvert qu’un paramètre de contrôle physique important est la taille du film épais \(d_2\). Nous avons donc amélioré le modèle avec une taille finie. Avec cette correction, le modèle donne la bonne tendance mais reste une surestimation, ce qui pourrait s’expliquer par le fait que le système n’est pas stationnaire ou que les conditions aux bords sont mal modélisées.

La question du régime non linéaire de l’instabilité nécessite de nouvelles études expérimentales et théoriques. Durant les derniers mois de mon doctorat, j’ai travaillé avec un étudiant en stage sur une configuration expérimentale permettant de suivre la trajectoire d’un patch de film épais créé pendant l’instabilité et de mesurer sa vitesse. Trois forces agissent sur le patch: la force visqueuse, le poids et la force de flottabilité. En les équilibrant, nous pouvons estimer la valeur de la viscosité de surface. Nous avons essayé d’améliorer la configuration afin de l’utiliser comme rhéomètre pour la viscosité de cisaillement à l’interface.

Au chapitre 5, nous avons donné les premières idées sur le couplage dynamique entre films de savon dans un échantillon de mousse à l’échelle millimétrique. La plus grande contribution à cette question a été la conception et les premiers tests du dispositif. Nous avons clairement compris qu’il existait un transfert d’interface entre les ménisques. La condition principale pour les écoulements locaux dans les mousses est la vitesse de transfert et ses premières mesures ont déjà été effectuées [10] grâce à une configuration bien testée et développée. Nous avons également breveté cette invention en tant que rhéomètre pour films liquides.
Bibliography


Titre : Déformation de films de savon et instabilités

Mots clés : films liquides, mousses, viscosité, instabilité, dissipation

Résumé : Les mousses liquides soumises à du cisaillement présentent une très grande viscosité, mais l'origine locale de la dissipation se produisant pendant cette déformation est encore mal comprise. Dans le but d'apporter quelques éléments de réponses à cette importante question ouverte, notre travail décrit le comportement observé sur quelques films connectés lorsqu'une déformation leur est appliquée.

Nous avons créé un montage permettant de fabriquer un pattern élémentaire de mousse, et de modifier la taille de chaque film en contrôlant la géométrie du cadre qui le supporte. Ce montage original, auquel s'ajoute une combinaison d'appareils optiques, nous permet de révéler les processus se produisant dans le film, notamment la compétition entre son allongement ou compression, et l'extraction d'un nouveau film depuis les ménisques raccordant les films. Nous montrons de plus que cette compétition dynamique dans un film donné est affectée par la déformation de ses premiers et seconds films voisins.

La géométrie particulière du montage nous a également permis de découvrir et de décrire pour la première fois une instabilité gravitationnelle se produisant lorsqu'un film épais se situe au-dessus d'un film plus mince. Nous avons mesuré la longueur d'onde de l'instabilité et l'avons comparée à des prédictions théoriques en régime linéaire.

Ces différents écoulments aident la distribution d'épaisseur dans le film, et peuvent jouer un rôle important sur la viscosité ou sur la stabilité des mousses 3D.

Finalement, le montage utilisé pourra s'avérer utile à l'avenir comme rhéomètre de films liquides.

Title : Deformations and instabilities of soap films

Keywords : liquid films, foams, viscosity, instability, dissipation

Abstract : Liquid foams under shearing exhibit a large effective viscosity, and the understanding of the local origin of the dissipation occurring during deformation is unknown. In the aim to contribute to this important open problem, we tried to describe the behavior of a few connected films under deformation.

We created a setup allowing to make an elementary foam sample and to modify each film size by controlling the shape of the deformable frame supporting the films. This original setup together with a combination of optical devices allowed us to reveal processes happening in the film, and especially the competition between film stretch or compression, and extraction of a new film from the menisci connecting the films. Importantly, we show that this dynamical competition in a given film is affected by the deformation of its first and even second neighbors.

The unique geometry of the setup gave us the opportunity to discover and describe for the first time a gravitational instability which takes place when a thicker film is on top of a thinner one. We measured the wavelength and compared it to theoretical predictions in the linear regime.

These different flows affect the thickness distribution, and may thus play an important role in the viscosity or in the stability of 3D foams.

As a perspective, the designed setup could prove to be useful as a liquid film rheometer.